Abstracts of the

10th Slovenian Conference on Graph Theory

Kranjska Gora, Slovenia, June 18 – 24, 2023

June 2023

Welcome

We live in turbulent times, where events that seemed unimaginable at our last meeting four years ago in Bled have become reality. Even as the horsemen of the apocalypse continue to ravage the world, the beauty and depth of recent advances in mathematics and science give us hope for the future. But these are also exciting times, where science is catching up with science fiction, and tools that were once deemed unattainable are now within reach for anyone with internet access. Have we developed a remarkable tool that will help us transcend our flaws, or have we created a monster that may eventually lead to our destruction? Only time will tell. For now, let us marvel at the work of ChatGPT and discover how they will greet us here. What follows is a text generated by ChatGPT, an AI language model developed by OpenAI, when asked to write a welcome text for the conference.

"Welcome to the 10th Slovenian Conference on graph theory (SiCGT23), also known as the Bled conference, held this year in the enchanting town of Kranjska Gora, Slovenia! We are thrilled to mark a significant milestone in the conference's history as we celebrate its 10th edition. Originating back in 1985 with its inaugural meeting in Dubrovnik, the Bled conference has been a cornerstone event for the graph theory community. Since 1991, it has been held every four years, captivating the imagination of researchers and practitioners alike. Over the years, the conference has grown in stature and global recognition, cementing its place as one of the largest gatherings of its kind in the world.

We are delighted to welcome over 300 participants from around the globe to SiCGT23. As we convene in the idyllic setting of Kranjska Gora, nestled amidst the breathtaking Slovenian Alps, we embark on a journey that combines the rich heritage of the Bled conference with the visionary spirit of the Slovenian Conference on graph theory. SiCGT23 promises to be an extraordinary event, featuring an exceptional lineup of talks, presentations, and discussions that push the boundaries of graph theory. Renowned speakers will share their insights, showcasing groundbreaking research, novel methodologies, and interdisciplinary applications that continue to shape the field.

Beyond the intellectual exchange, SiCGT23 provides a nurturing environment for forging new connections, fostering collaborations, and strengthening the global graph theory community. Against the backdrop of Kranjska Gora's natural beauty, we encourage participants to engage in lively discussions, share ideas, and build lasting friendships that will extend far beyond the conference.

We express our sincere gratitude to all the participants, speakers, and organizers who have contributed to the success of the Bled conference over the past three decades. Your unwavering commitment and scholarly enthusiasm have been instrumental in elevating the conference to its esteemed status today.

As we embark on this landmark edition, let us honor the legacy of the Bled conference while embracing the spirit of innovation, discovery, and camaraderie that defines SiCGT23. May this conference be a catalyst for transformative breakthroughs, collaborative partnerships, and cherished memories.

Once again, a warm welcome to SiCGT23, the 10th Slovenian Conference on graph theory, also known as the Bled conference. We wish you a remarkable and fulfilling experience that will shape the future of graph theory and inspire generations to come."

Kranjska Gora, June 2023 — *the Scientific Committee of SiCGT'23*

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SICGT23 – 10th Slovenian International Conference on Graph Theory
Kranjska Gora, Slovenia, June 18 – 24, 2023

Organized by:
UL FMF (*University of Ljubljana, Faculty of Mathematics and Physics*),

In Collaboration with:

IMFM (*Institute of Mathematics, Physics and Mechanics*)

UP FAMNIT (*University of Primorska, Faculty of Mathematics, Natural Sciences and Information Technologies*),

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Vizing's Conjecture: different approaches

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Vizing's Conjecture (1968) is one of the central conjectures in domination theory. It states that every two graphs G and H satisfy the inequality

$$\gamma(G \,\Box\, H) \geq \gamma(G)\gamma(H),$$

where $\gamma(G)$, $\gamma(H)$, and $\gamma(G \,\Box\, H)$ denote the domination number of G, H, and the Cartesian product $G \,\Box\, H$, respectively.

We survey some earlier results and approaches from the related literature and present new ideas and results.

Flows, coloring, and homology

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The existence of a 3-coloring of a graph on an orientable surface can be easily seen to be equivalent to the existence of a nowhere-zero 3-flow in its dual subject to additional homological conditions, and further related to the presence of integer points in certain polytopes of bounded dimension. We detail this connection and present its consequences, such as an unexpected connection between colorability of graphs on surfaces and a long-standing conjecture concerning the width of hollow polytopes, improved bounds on the edgewidth of non-3-colorable triangle-free graphs, and a practical algorithm for 3-precoloring-extension in near-quadrangulations.

Classification problems in Finite Geometry

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Mathematicians have always been intrigued by the challenge to obtain a complete list of "inequivalent" objects satisfying a given number of axioms. The equivalence is usually determined by a group action on the set of objects at hand. Throughout the history of mathematics, many beautiful classification results have been obtained, and some of these took centuries to prove.

In Combinatorics classification results naturally appear in all types of counting problems. For example, in Graph Theory where the subject is sometimes referred to as Graph Enumeration. Although solutions to such problems do not necessary comprise a full classification, they are often a combination of smaller classification results on particular subproblems.

In this talk, I will focus on classification problems in Finite Geometry, and will give an overview of some of the most celebrated classifications, a bucket list of (longstanding) open problems, and recent progress on some of these.

Intersection Density of Permutation Groups

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Two permutations π and σ are **intersecting** if there is an i with $\pi(i) = \sigma(i)$. A set of permutations is called intersecting if any two permutations in the set are intersecting. For any permutation group we can ask "What is the largest intersecting set of permutations?".

The stabilizer of any point is an intersecting set, so we can measure any intersecting set in a group by comparing its size to the size of a stabilizer in the group. Specifically, for any transitive permutation group the **intersection density** is defined to be the ratio of the size of the largest intersecting set in the group, to the size of the stabilizer of a point. If the intersection density of a group is 1, then the stabilizer of a point is an intersecting set of maximum size; such groups are said to have the **Erdős-Ko-Rado property**. There have been many results proving specific groups have intersection density 1, including all 2-transitive groups. On the other hand, there are examples of groups that have a large intersection density. I will present some of these examples and some bounds on the intersection density.

One effective way to determine the intersection density of a group is to build a graph whose vertices are the elements of the group and the edges are defined so that the cocliques (or the independent sets) in the graph are exactly the intersecting sets in the group. This graph is called the **derangement graph** for the group. The eigenvalues of these graphs can be found using the representation theory of the group, and using tools from algebraic graph theory these eigenvalues can be used to bound the size of a coclique. For a surprising number of groups this gives the exact value of the intersection density.

I will end with some example the groups where these methods don't seem to work and a new approach is needed.

This is joint work with A. Sarobidy Razafimahatratra and Pablo Spiga.

On Hamilton cycles in highly symmetric graphs

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In 1970 Lovász conjectured that every connected vertex-transitive graph has a Hamilton cycle, apart from five exceptional graphs, one of them being the infamous Petersen graph. This problem received a lot of attention, and it has far-ranging connections to algebra, algorithms, geometry, etc. The question turns out to be surprisingly difficult even for vertex-transitive graphs defined by explicit constructions, e.g., Cayley graphs for various groups and generators. Another plentiful source of vertex-transitive graphs are intersecting set systems. One example for this are Kneser graphs $K(n, k)$, whose vertices are all k-element subsets of an n-element set, and the edges connect disjoint sets. In this talk I present our line of work that settles Lovász' conjecture for all known graphs defined by intersecting set systems. In particular, we show that all Kneser graphs $K(n, k)$ admit a Hamilton cycle, except the Petersen graph $K(5, 2)$.

Recent Results on the Irregularity and the Mostar Index of Graphs

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In 1997 Albertson defined the *irregularity* of a graph G as

$$irr(G) = \sum_{uv \in E(G)} |d_G(u) - d_G(v)|,$$

and in 2018 Došlić, Martinjak, Škrekovski, Tipurić Spužević, and Zubac defined the *Mostar index* of G as

$$Mo(G) = \sum_{uv \in E(G)} |n_G(u,v) - n_G(v,u)|,$$

where, for an edge uv of G, the term $n_G(u,v)$ denotes the number of vertices of G that have a smaller distance in G to u than to v.

In this talk we present some recent results concerning these two graph parameters that have both attracted considerable attention. We present a best possible bound on the irregularity of a graph G of maximum degree Δ in terms of its order n and size m. More precisely, for $d = \left\lfloor \frac{\Delta m}{\Delta n - m} \right\rfloor$, we show

$$irr(G) \leq d(d+1)n + \frac{1}{\Delta}\left(\Delta^2 - (2d+1)\Delta - d^2 - d\right)m.$$

Došlić et al. posed two conjectures concerning the maximum value of the Mostar index of general graphs and bipartite graphs in terms of their order. Contributing to their conjectures we show that the Mostar index of a graph of order n is at most $\frac{5}{24}(1 + o(1))n^3$, that the Mostar index of a bipartite graph of order n is at most $\frac{\sqrt{3}}{18}n^3$, and that the Mostar index of a split graph of order n is at most $\frac{4}{27}n^3$.

The talk is based on joined work with Štefko Miklavič, Johannes Pardey, and Florian Werner; details can be found in the three arXiv posts 2210.03399, 2211.06682, and 2303.12632.

Every one a winner: an introduction to orderly algorithms

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The computer generation of complete catalogues of pairwise non-isomorphic graphs and other combinatorial objects (designs, geometries, matroids etc.) is a fundamental part of research in combinatorics. However the naive method of removing isomorphs—namely testing each newly constructed graph against each previously-constructed graph in the growing catalogue is possible only for tiny catalogues. In a seminal paper in the 1970s entitled *"Every one a winner; or how to avoid isomorphism search when cataloguing combinatorial configurations"*, Ron Read outlined an algorithm—that he called an *orderly algorithm*—for constructing isomorph-free catalogues without explicitly testing any pairs of graphs. In this talk, I will explain in some detail the principles behind Read's orderly algorithm, how its main weaknesses were addressed by its spiritual successor, Brendan McKay's canonical construction path algorithm, before finishing with some examples of the use of this latter algorithm.

From synchronization, to permutation groups, to graphs

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An automaton is synchronizing if there is a word in the transitions which sends all states of the automaton to a single state. The scope of this talk is to present a series of natural definitions in the theory of permutation groups, arising from the oldest problem in automata theory, that is, Černý conjecture on synchronizing automata. These definitions have remarkable and unexpected connections with a number of seemingly unrelated areas. Most of these connections are concerned with finite geometry, with complete mappings and with finite graphs. A number of natural problems in permutation groups are motivated by this work and we present some of the most intriguing.

Association schemes with many fusions

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Association schemes are colorings of the edges of the complete graph satisfying many combinatorial regularity conditions. Because of these conditions, the adjacency matrices of the colors generate a nice algebra, the Bose-Mesner algebra. Important, and in some sense extremal examples are colorings given by distance in a distance-regular graph, the so-called P-polynomial schemes. Dual to this are Q-polynomial schemes. Typical of such schemes is that they have few fusion schemes, where we speak of a fusion scheme of a scheme if joining some of the colors gives rise to another association scheme.

In this talk, we will focus on association schemes with many fusions, in particular amorphic schemes. The latter are extreme in the sense that no matter how you join colors, they always give rise to a fusion. We will give an overview of results on such schemes, including some new ones. Fundamental to these results are good old strongly regular graphs and linear algebra.

Proof of the Clustered Hadwiger Conjecture

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Hadwiger famously conjectured that every K_h-minor-free graph is properly $(h-1)$-colourable. We prove the following improper analogue of Hadwiger's Conjecture: for fixed h, every K_h-minor-free graph is $(h-1)$-colourable with monochromatic components of bounded size. The number of colours is best possible regardless of the size of monochromatic components. This solves an open problem of Edwards, Kang, Kim, Oum and Seymour [*SIAM J. Disc. Math.* 2015], and concludes a line of research initiated in 2007. Similarly, for fixed $t \geqslant s$, we show that every $K_{s,t}$-minor-free graph is $(s+1)$-colourable with monochromatic components of bounded size. The number of colours is best possible, solving an open problem of van de Heuvel and Wood [*J. London Math. Soc.* 2018]. We actually prove a single theorem from which both of the above results are immediate corollaries. For an excluded apex minor, we strengthen the result as follows: for fixed $t \geqslant s \geqslant 3$, and for any fixed apex graph X, every $K_{s,t}$-subgraph-free

X-minor-free graph is $(s+1)$-colourable with monochromatic components of bounded size. The number of colours is again best possible. This is joint work with Vida Dujmović, Louis Esperet and Pat Morin.

Directed max-cut and some generalizations

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We will discuss some results on directed max-cut and their generalizations. That is, for a digraph D, find a partition (X, Y) of $V(D)$ containing as many arcs from X to Y as possible. We first look at the arc-weighted case and present a few baic bounds. We then consider the case when D is acyclic and all weights are at least one and show the main ideas of how to prove that the weight of a maximum weighted cut is always at least $W/4 + c_1 \cdot W^{0.6}$, where $c_1 = 1/24$ and W is the sum of all weights in the digraph. We also give an infinite class of digraphs where no cut has weight more than $W/4 + c_2 \cdot W^{0.75}$ for some constant c_2. We can generalize the results to digraphs with circumference (ie the length of a longest cycle) bounded by a constant. It is an open problem to close the gap between 0.6 and 0.75.

In the second part of the talk we will generalize max-cut to the following case. Let D be a digraph, such that for each arc $a \in A(D)$ we are given values $xx(a)$, $xy(a)$, $yx(a)$ and $yy(a)$. We want to find a partition (X, Y) of $V(D)$ that maximizes $\sum_{a \in A(D)} val(a)$, where

$$val(uv) = \begin{cases} xx(uv) & \text{if } u, v \in X \\ xy(uv) & \text{if } u \in X \text{ and } v \in X \\ yx(uv) & \text{if } u \in Y \text{ and } v \in X \\ yy(uv) & \text{if } u, v \in Y \end{cases}$$

If the classes of allowed values $(xx(a), xy(a), yx(a), yy(a))$ (where multiples thereof are also allowed) are given, then we can determine if the problem is polynomial or NP-hard in all cases. That is, we obtain a dichotomy of this problem. We will in this talk present this result together with some of its applications. Note that if $(xx(a), xy(a), yx(a), yy(a)) = (0, 1, 0, 0)$ for all arcs a, then the above problem is equivalent to the normal max-cut in digraphs.

General Session

On generalized grid torus

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Most of the parallel computers have been designed using interconnection networks like meshes, tori and hypercubes. Tori networks have a rich history that spans many decades. Tori networks are very attractive for various practical reasons such as packing, modularity, cost and scalabilty. Due to its topological advances, Tori interconnection networks have been the most popular topology in designing different technology from Cray, HP and IBM [5,7]. Twisted torus, rectangular twisted torus, doubly twisted torus and spiral torus are some of the grid based tori architectures that are widely studied in literature [1,2,3,4]. The idea of twisting a 2-dimensional tori was developed to obtain architectural benefits like network symmetry [6]. In this paper, a generalized family of grid based tori namely g-Tori is defined.

A g-Torus denoted $gT_{m,n,k,l}$ where $0 \leq k \leq m-1, 0 \leq l \leq n-1$ is a $P_mP_n$grid of mn vertices arranged with labels (x,y) such that $0 \leq x \leq m-1$ and $0 \leq y \leq n-1$. The wraparound edges are defined as:

- $(x,0)$ is adjacent to $(x \oplus_m k, n-1)$, $\forall x$,$0 \leq x \leq m-1$
- $(0,y)$ is adjacent to $(m-1, y \oplus_n l)$, $\forall y$,$0 \leq y \leq n-1$

The architecture and design of this family are examined to provide a global view. Various topological properties of g-Tori are investigated. The Hamiltonian property and Hamiltonian cycle decomposition of g-Tori are studied. The planarity and crossing number of this family on torus surface (S_1, surface of genus one) are analyzed. The isomorphic pairs and also provide equivalent g-Tori for certain existing tori are characterized.

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Uniform Monitoring Set Problem in Power of Cycles

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The edge monitoring is considered as a method used to detect misbehavior nodes in wireless sensor networks. We consider the graph theoretical representation of this problem such that each node is represented by a vertex and the communication link by an edge. Given a graph $G = (V, E)$, the problem is to seek a subset of monitors S such that every edge e in the graph is monitored by at least one vertex in S (A vertex v can monitor an edge e if both extremities together with v form a triangle in the graph). In this work, we give a particular interest on 1-uniform monitoring set problem where all edges need to be monitored by at least one monitor. We study the 1-UNIFORM MONITORING SET problem on cycle power C_n^k and find the minimum 1-uniform monitoring set.

A fruitful approach for signed graphs

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A signed graph can be described as an undirected graph whose edges are assigned a $+$ or $-$ sign; the edges are said to be *positive* or *negative* according to whether they have $+$ or $-$ sign, respectively.

Signed graphs have been introduced by Harary in [1] to develop and formalize a psychological theory proposed by Heider [2] for analyzing the network of relations in a group of people. In particular, balanced signed graphs are privileged worlds where stability reigns, thanks to two disjoint subgraphs each one internally positive, while externally negative. Balance is actually hard to find in ordinary life, several relaxations are studied in the literature in order to better suit reality. In this context, we introduce two novel discrete structures: fruitful coverings and antagonism colorings.

References:

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[2] F. Heider, *The Psychology of Interpersonal Relations*, Wiley, New York, 1958.

Twins and Semitwins in Digraphs

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Let D be a digraph, $V(D)$ and $A(D)$ will denote the sets of vertices and arcs of D, respectively. Given $u, v \in V(D)$ we say that they are semitwins if $N^-(u) = N^-(v)$ or $N^+(u) = N^+(v)$ where $N^-(v) = \{x \in V(D) : (x, v) \in A(D)\}$ and $N^+(v) = \{x \in V(D) : (v, x) \in A(D)\}$. Also, we say that u and v are twins if $N^-(u) = N^-(v)$ and $N^+(u) = N^+(v)$. A digraph D is a semitwin digraph if every pair of adjacent vertices in D are semitwins. In this talk we prove that if D is a semitwin strong digraph then D is vertex-pancyclic. Also, we characterized the semitwin strong digraphs and the semitwin connected digraphs which are not strong connected.

On the structure of quaternion rings

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The ring of real quaternions $\mathbb{H} = \mathbb{R} \oplus \mathbb{R}i \oplus \mathbb{R}j \oplus \mathbb{R}k =: H(\mathbb{R})$ was discovered by William Rowan Hamilton (1805-1865) in 1843 as an extension of the complex numbers into four dimensions. Algebraically speaking, $\mathbb{H}$ forms a division algebra over $\mathbb{R}$ of dimension four. Hamilton expanded this detection to include applications in the area of physics and in 1853 published Lectures "On Quaternions" and in 1866 "Elements of Quaternions". Hamilton's quaternions have many applications other than in physics. They are extensively used in computer graphics to describe motion in 3-space, and more recently, they have been used in multiple antennae communications systems. Since the quaternions were the first discovered noncommutative division ring, an investigation of their properties and construction became the basis of this study.

The main purpose of this talk is to describe some algebraic structures of the quaternion ring $\mathcal{R} = H(\mathcal{S})$ for an arbitrary unital ring $\mathcal{S}$ with $2^{-1} \in \mathcal{S}$. Also, some properties of quaternion ring over the ring of integers $\mathbb{Z}$ and the ring $\mathbb{Z}_{p^n}$, where p is a prime number and $n \geq 1$, have been considered. It is shown that in most cases the structure of center and ideals of the quaternion rings and those of matrix rings are similar. This talk is organized as follows. Some preliminaries are given in Section 1. The structure of ideals and some radicals of $H(\mathcal{S})$ are presented in Section 2, where its basic properties are given too. In Section 3, we show that under some mild conditions on $\mathcal{S}$, the rings $H(\mathcal{S})$ and $M_2(\mathcal{S})$ are isomorphic. Section 4 deals with some common and noncommon properties of $H(\mathcal{S})$ and $\mathcal{S}$.

Neumaier graphs

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A *Neumaier graph* is an edge-regular graph with a regular clique. A lot of strongly regular graphs (but not all of them) are indeed Neumaier, but in 1981 it was asked whether there are Neumaier graphs that are not strongly regular [3]. This question was only solved recently by Greaves and Koolen [2], so now we know there are so-called *strictly* Neumaier graphs.

In this talk I will discuss several new results on (strictly) Neumaier graphs, including bounds on the parameters and (non)-existence results. I will focus on a new construction (involving Cayley graphs) producing an infinite number of strictly Neumaier graphs, but I will also discuss a new Neumaier graph arising from a Latin square. This talk is based on joint research with A. Abiad, W. Castryck, J.H. Koolen and S. Zeijlemaker [1].

References:

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Toll convexity

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A walk W between two non-adjacent vertices in a graph G is called tolled if the first vertex of W is among vertices from W adjacent only to the second vertex of W, and the last vertex of W is among vertices from W adjacent only to the second-last vertex of W. In the resulting interval convexity, a set $S \subseteq V(G)$ is toll convex if for any two non-adjacent vertices $x, y \in S$ any vertex in a tolled walk between x and y is also in S. Using toll convexity, a new characterization of interval graphs was proved. Toll convexity give rise to some well-known types of invariants, such as t-convexity number, toll number, toll hull number, etc. In this talk we consider these invariants in different graph families and show that in many graph classes the toll number of a graph G can be bounded above by a constant plus the number of extreme vertices of G.

House of Graphs 2.0: a database of interesting graphs and more

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In 2012 we announced the House of Graphs (https://houseofgraphs.org/), which was a new database of graphs. The House of Graphs hosts complete lists of graphs of various graph classes, but its main feature is a searchable database of so called "interesting" graphs, which includes graphs that already occurred as extremal graphs or as counterexamples to conjectures. An important aspect of this database is that it can be extended by users of the website.

Over the years, several new features and graph invariants were added to the House of Graphs and users uploaded many interesting graphs to the website. But as the development of the original House of Graphs website started in 2010, the underlying frameworks and technologies of the website became outdated. This is why we recently completely rebuilt the House of Graphs using modern frameworks to build a maintainable and expandable web application that is future-proof. On top of this, several new functionalities were added to improve the application and the user experience.

In this talk we will present the House of Graphs and highlight the changes and new features of the new website. We will also demonstrate how users can perform queries on this database and how they can add new interesting graphs to it.

Bootstrap percolation in strong products of graphs

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Given a graph G and assuming that some vertices of G are infected, the r-neighbor bootstrap percolation rule makes an uninfected vertex v infected if v has at least r infected neighbors. The r-percolation number, $m(G, r)$, of G is the minimum cardinality of a set of initially infected vertices in G such that after continuously performing the r-neighbor bootstrap percolation rule each vertex of G eventually becomes infected. In this talk, we consider percolation numbers of strong products of graphs. If G is the strong product of k connected graphs, we prove that $m(G, r) = r$ as soon as $r \leq 2^{k-1}$ and as a dichotomy, we present a family of strong products of k connected graphs with the $(2^{k-1} + 1)$-percolation number arbitrarily large. Among others we also provide a characterization of strong prisms $G \boxtimes K_2$ for which $m(G \boxtimes K_2, 3) = 3$ and consider extensions to strong products of two-way infinite paths.

On a generalization of median graphs: k-median graphs

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Median graphs are connected graphs in which for all three vertices there is a unique vertex that belongs to shortest paths between each pair of these three vertices. To be more formal, a graph G is a median graph if, for all $\mu, u, v \in V(G)$, it holds that $|I(\mu, u) \cap I(\mu, v) \cap I(u, v)| = 1$ where $I(x, y)$ denotes the set of all vertices that lie on shortest paths connecting x and y.

In this talk, we consider a natural generalization of median graphs, called k-median graphs. A graph G is a k-median graph, if there are k vertices $\mu_1, \dots, \mu_k \in V(G)$ such that, for all $u, v \in V(G)$, it holds that $|I(\mu_i, u) \cap I(\mu_i, v) \cap I(u, v)| = 1$, $1 \leq i \leq k$. By definition, every median graph with n vertices is an n-median graph. We provide several characterizations of k-median graphs that, in turn, are used to provide many novel characterizations of median graphs.

The results are available at https://arxiv.org/abs/2304.06453

Finite and infinite hierarchical product graphs

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In 1974 Schwenk introduced a binary operation of graphs to study their spectra. This operation was generalized by Godsil and McKay in 1978 for the investigation of the spectrum of trees, and then rediscovered and called hierarchical product by Barrière, Comellas et al. in 2009. In the same year Barriere, Dalfó et al. introduced the generalized hierarchical product. Still, the study of the spectrum of the resulting graphs was the motivating or primary motivating factor.

The hierarchical and the generalized hierarchical product are not commutative, but associative under certain conditions. For the associative case unique prime factorization with respect to both products was claimed in 2017 by Anderson et al. for finite connected graphs.

We provide a new proof for unique prime factorization of finite connected graphs with respect to the hierarchical product and extend the result to the non-associative case, in which the factorizations may become finer. On the way we also derive a straightforward linear algorithm for the prime factorization of finite trees with respect to the hierarchical product.

For infinite graphs unique prime factorization does not hold in general, but it holds for compact connected graphs, that is, for connected graphs without rays. For this part we invoke results of Schmidt from 1983 about the rank of rayless graphs.

For the generalized hierarchical product of connected finite graphs we provide counterexamples to unique prime factorization.

Vertex colored digraphs explained by phylogenetic trees

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Best match graphs (BMGs) are a family of properly vertex colored digraphs explained by a phylogenetic tree. They have been studied in mathematical phylogenetics in the last ten years because they model the closest gene across different species. Quasi-best match graphs (qBMGs) form a hereditary family of properly vertex colored digraphs, which has been recently introduced to generalize the notion of BMG. In this talk, we will start by seeing some properties of BMGs, including a characterization in terms of triples and a polynomial time algorithm to build the associated phylogenetic tree. Some of these properties also hold for qBMGs; however, whether this family of graphs also has properties that fit well in structural graph theory has not been completely investigated yet.

Finding a Hamiltonian cycle using the Chebyshev polynomials

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The characteristic polynomial of the adjacency (or Laplacian) matrix of an undirected graph formed by a single Hamiltonian cycle is related to some Chebyshev polynomial of the first kind. We use the properties of Chebyshev polynomials and present the algorithm consisting in finding a Hamiltonian cycle by minimization of an appropriately chosen functional.

The second largest eigenvalue of normal Cayley graphs on symmetric groups generated by cycles

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Aldous' Spectral Gap Conjecture states that the second largest eigenvalue of each connected Cayley graph on the symmetric group S_n with respect to a set of transpositions is attained by the standard representation of S_n. This celebrated conjecture, which was proposed in 1992 and completely proved in 2010, has inspired much interest in determining the second largest eigenvalue of Cayley graphs on S_n. In this talk, we focus on the normal Cayley graphs $\mathrm{Cay}(S_n, C(n, I))$ on the symmetric group S_n, where $I \subseteq \{2, 3, \dots, n\}$ and $C(n, I)$ is the set of all cycles in S_n with length in I. We show that the strictly second largest eigenvalue of $\mathrm{Cay}(S_n, C(n, I))$ can only be achieved by at most four irreducible representations of S_n, and we determine further the multiplicity of this eigenvalue in several special cases. As a corollary, in the case when I contains neither $n-1$ nor n we know exactly when $\mathrm{Cay}(S_n, C(n, I))$ has the Aldous property, namely the strictly second largest eigenvalue is attained by the standard representation of S_n, and we obtain that $\mathrm{Cay}(S_n, C(n, I))$ does not have the Aldous property whenever $n \in I$. As another corollary of our main results, we prove a recent conjecture on the second largest eigenvalue of $\mathrm{Cay}(S_n, C(n, \{k\}))$ where $2 \leq k \leq n-2$.

Clique Dynamics of Finite or Infinite Locally Cyclic Graphs with $\delta \geq 6$

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We prove that the clique graph operator k is divergent on a (not necessarily finite) locally cyclic graph G (i. e. $N_G(v)$ is a circle for every vertex v) with minimum degree $\delta(G) \geq 6$ if and only if the universal cover of G contains arbitrarily large triangular-shaped subgraphs. For finite G, this is equivalent to G being 6-regular. The clique graph G of a graph G has the maximal complete subgraphs of G as vertices and its edges are given by non-empty intersections. The $(n+1)$-st iterated clique graph is inductively defined as the clique graph of the n-th iterated clique graph. If all iterated clique graphs of G are pairwise non-isomorphic, the graph G is called k-divergent; otherwise, it is k-convergent.

Locally cyclic graphs with $\delta \geq 6$ which induce simply connected simplicial surfaces are isomorphic to their universal covers. On this graph class, we prove our claim by explicit construction of the iterated clique graphs.

After that, we show that locally cyclic graphs with $\delta \geq 6$ are k-convergent if and only if their universal covers are k-convergent. This way, we can drop the condition of simple connectivity.

This talk is based on joint work with Markus Baumeister and Martin Winter.

Scarce and frequent cycles in planar graphs with minimum degree 4

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An n-cycle C_n is called *frequent* in a graph family $\mathcal{G}$ if it is a subgraph of each graph $G \in \mathcal{G}$ provided G has enough vertices. Non-frequent cycle C_n is called *scarce*; this means that there exists an infinite sequence $\{G_i\}_{i=1}^{\infty}$ of graphs from $\mathcal{G}$ such that no G_i contains C_n as a subgraph. In the talk, we describe various construction showing that almost every cycle is scarce in the class of 3-connected planar graphs with minimum degree at least 4 and minimum edge weight at least 9. Moreover, we show that C_8 is frequent in the class of plane graphs with minimum degree 5.

Šoltés's type problems for resistance distances in graphs

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In 1991, Šoltés formulated the problem of finding all graphs that preserve Wiener index when arbitrary vertex is removed. We consider a problem of Šoltés's type for resistance distances in graphs. Instead of Wiener index, we consider Kirchhoff's index, which is the sum of the resistance distances between all pairs of vertices in the graph, and we pose the K-Šoltés's problem:

K-Šoltés's problem: Find all graphs G such that for every vertex $v \in V(G)$ the equality $Kf(G) = Kf(G - v)$ holds.

We discover only one graph with desired property. Next, we study a relaxed version of K-Šoltés's problem and consider $n-$vertex graphs for which the Kirchhoff index does not change when at least one vertex is removed. All such graphs with $n \leq 8$ vertices are found. An infinite family of graphs with half good vertices is constructed. Furthermore, an infinite family of graphs with the proportion of good vertices tending to $2/3$ is built.

On the properties of Villarceau Torus

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Spiral torus models are used to define the electron and positron, which are then used to explain the nature of electric and magnetic fields. Villarceau torus $\mathrm{VT}(r, s)$ is a graph theory model of spiral torus which is popular in Physics. We observe some interesting properties of Villarceau torus $\mathrm{VT}(r, s)$.

1. When $r = s$, some cycles of $\mathrm{VT}(r, s)$ model Villarceau circles.
2. When $s \geq r$, some cycles model toroidal helices.
3. When $\gcd(r, s) = 1$, some cycles model double-stranded DNA molecules which are also called double helix DNA models.

We also prove that Villarceau torus does not admit any convex cut which provides an efficient technique to find topological indices such as Wiener Index. Further, we extend the convex cut method to graphs which do not admit any convex cuts. Finally, we derive elegant routing algorithm for Villarceau torus and demonstrate that Villarceau torus is a better parallel architecture than similar architectures such as ring torus.

On local structure of planar graphs with degree four

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One of the typical approaches in studying the structure of graphs (not necessarily planar or embedded) is to study the local properties of graphs belonging to specific subclasses of a given "main" class of graphs, based on existing knowledge on the structure of graphs from this wider superclass.

In the talk, we illustrate this approach in the context of selected solved/open problems concerning the existence vs. exclusion of cycles of prescribed lengths and the types of small degree vertices in planar graphs of minimum degree at least 4, particularly for the subclass of knots (4-regular planar graphs with Eulerian geodesic trail; in simplified way, they are projections of certain three-dimensional knots onto the plane).

As a partial new tool for constructing knots that prove the optimality of some structural results inherited from the superclass of planar graphs of minimum degree at least 4, Lebesgue's scarfs will be used.

The analogous questions on the structure will also be addressed for other specific 4-regular planar graphs (such as Grötzsch-Sachs graphs or Venn diagrams).

Extendability of Perfect Matchings to a Hamiltonian Cycle in the Cartesian Product of Graphs

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Let G be a graph of even order, and consider K_G the complete graph on the same vertex set as G. A perfect matching of K_G is called a *pairing of* G. If for every pairing M of G it is possible to find a perfect matching N of G such that $M \cup N$ is a Hamiltonian cycle of K_G, then G is said to have the *Pairing Hamiltonian Property*, or PH-property, for short. Fink proved in 2007 that, for every $n > 1$, the hypercube Q_n has the PH-property. Here, we extend his result by proving that if G has the PH-property then also the Cartesian product $G \Box K_2$ has the PH-property. Moreover, if G is a connected graph, we prove that there exists $d_0(G) \geq 2$ such that $G \Box Q_d$ has the PH-property, for every $d \geq d_0(G)$.

Anti-Ramsey type problems for maximal stars and long cycles

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We give an overview of known results on the problem of determining the maximum number of colors $\mathcal{K}_f(G)$ in an M_f-edge coloring, i.e., a coloring in which the number of colors used on edges incident to each vertex v is at most $f(v)$, where f is a mapping from $V(G)$ to positive integers. We provide several bounds on $\mathcal{K}_f(G)$ using dominating sets in graphs, present some graphs achieving these bounds, and determine exact values of $\mathcal{K}_f(G)$ for graphs of special classes. Motivated by the results on M_f-edge colorings and their special cases – M_i-edge colorings, we define a new problem: determine the maximum number of colors $\mathcal{K}_i^\circ(G)$ in an edge coloring with no i^+-colored cycle. We prove a general lower bound on $\mathcal{K}_i^\circ(G)$ and present exact values of it in the case of small values of i.

On the number of maximal independent sets: From Moon-Moser to Hujter-Tuza

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We connect two classical results in extremal graph theory concerning the number of maximal independent sets. The maximum number $\mathrm{mis}(n)$ of maximal independent sets in an n-vertex graph was determined by Miller and Muller and independently by Moon and Moser. The maximum number $\mathrm{mis}_{\triangle}(n)$ of maximal independent sets in an n-vertex triangle-free graph was determined by Hujter and Tuza. We give a common generalization of these results by determining the maximum number $\mathrm{mis}_t(n)$ of maximal independent sets in an n-vertex graph containing no induced triangle matching of size $t+1$. This also improves a stability result of Kahn and Park on $\mathrm{mis}_{\triangle}(n)$. Our second result is a new (short) proof of a second stability result of Kahn and Park on the maximum number $\mathrm{mis}_{\triangle,t}(n)$ of maximal independent sets in n-vertex triangle-free graphs containing no induced matching of size $t+1$.

A conjecture on different central parts of binary trees

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Let Ω_n be the family of binary trees on n vertices obtained by identifying the root of an rgood binary tree with a vertex of maximum eccentricity of a binary caterpillar. In the paper [1], Smith et al. conjectured that among all binary trees on n vertices the pairwise distance between any two of center, centroid and subtree core is maximized by some member of the family Ω_n. We first obtain the rooted binary tree which minimizes the number of root-containing subtrees and then prove this conjecture (see [2]). We also identify certain binary trees which maximize these distances.

Breaking Symmetries in Graphs with Colors

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Symmetry in a graph G can be measured by investigating possible automorphisms of G. One way to do this is to color the vertices of G in such a way that only the trivial automorphism can preserve the color classes. If such a coloring exists with d colors, G is said to be d-*distinguishable*. The smallest d for which G is d-distinguishable is its *distinguishing number*. Another measure of symmetry is to consider subsets $S \subseteq V(G)$ such that the only automorphism that fixes the elements of S pointwise is the trivial automorphism. Such sets S are called *determining sets* or *fixing sets* for G and the *determining number* of G is the size of a smallest determining set.

Though the two parameters were introduced separately, relationships exist. In this talk, we'll investigate both of these parameters and their relationships.

Graphs of linear recurrences of length two

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We present a new family of graphs obtained from linear recurrences of length two with constant coefficients. They generalize Fibonacci cubes and preserve many of their interesting properties. In this talk, we elucidate their structure by presenting canonical decomposition and embedding into Fibonacci cubes and hypercubes. We also present results on their metric and enumerative properties such as radius, diameter and degree distribution.

On the minimum cut-sets of the power graph of a finite cyclic group

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Let Γ be a simple graph with vertex set V. A subset X of V is called a *cut-set* of Γ if the induced subgraph of Γ with vertex set $V \setminus X$ is disconnected. A cut-set X of Γ is called a *minimum cut-set* if $|X| \leq |Y|$ for any cut-set Y of Γ. The *power graph* of a finite group G, denoted by $\mathcal{P}(G)$, is the simple graph with vertex set G, in which two distinct vertices are adjacent if one of them is a power of the other. Given a finite group G, characterizing the minimum cut-sets of $\mathcal{P}(G)$ is an interesting problem. Among all finite groups of order n, the power graph $\mathcal{P}(C_n)$ of the cyclic group C_n of order n has the maximum number of edges and has the largest clique [2, 3]. It is thus expected that the size of a minimum cut-set of $\mathcal{P}(C_n)$ would be larger compared to the size of the minimum cut-set of the power graph of a noncyclic group of order n.

Let r denote the number of distinct prime divisors of n. For $r \leq 3$, the minimum cut-sets of $\mathcal{P}(C_n)$ are characterized in [1]. For $r \geq 4$, we identify certain cut-sets of $\mathcal{P}(C_n)$ such that any minimum cut-set of $\mathcal{P}(C_n)$ must be one of them. We explicitly describe the minimum cut-sets of $\mathcal{P}(C_n)$ when n is divisible by the square of its largest prime divisor. For other values of n, characterizing the minimum cut-sets of $\mathcal{P}(C_n)$ is still an open problem. The talk will be based on the results that appear in [4] and a few recent results.

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Observations on the Lovász θ-Function, Graph Capacity, Eigenvalues, and Strong Products

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This work provides new observations on the Lovász θ-function of graphs. These include a simple closed-form expression of that function for all strongly regular graphs, together with upper and lower bounds on that function for all regular graphs. These bounds are expressed in terms of the second-largest and smallest eigenvalues of the adjacency matrix of the regular graph, together with sufficient conditions for equalities (the upper bound is due to Lovász, followed by a new sufficient condition for its tightness). These results are shown to be useful in many ways, leading to the determination of the exact value of the Shannon capacity of various graphs, eigenvalue inequalities, and bounds on the clique and chromatic numbers of graphs. The utility of these bounds is exemplified, leading in some cases to an exact determination of the chromatic numbers of strong products or strong powers of graphs. The published version of this work as a journal paper appears in *Entropy*, vol. 25, no. 1, paper 104, pp. 1–40, January 2023 (https://doi.org/10.3390/e25010104).

Explicit Modular Decomposition

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The modular decomposition (MD) of an undirected graph G is a natural construction to capture key features of G in terms of a rooted labeled tree (T, t) whose vertices are labeled as "series" (1), "parallel" (0) or "prime" (p). If a graph G does not contain prime modules, then all structural information of G can be directly derived from its MD tree (T, t). As a consequence, many hard problems become polynomial-time solvable on graphs without prime modules (aka cographs), since the MD tree serves as a guide for algorithms to find efficient exact solutions (e.g. for optimal colorings, maximum clique, isomorphism test, ...).

However, the class of cographs is rather restricted. We introduce here the novel concept of explicit modular decomposition that aims at replacing "prime" vertices in the MD tree by suitable substructures to obtain 0/1-labeled networks (N, t). Understanding which graphs can be explained by which type of network does not only provide novel graph classes but is crucial to understand which hard problem can be solved on which graph class efficiently. We will mainly focus on graphs that can be explained by networks (N, t) whose bi-connected components are edge-disjoint. These graphs, called GaTEx, can be recognized in linear-time and are characterized by a set of 25 forbidden induced subgraphs. In particular, GaTEx graphs are closely related to many other well-known graph classes such as P_4-sparse and P_4-reducible graphs, weakly-chordal graphs, perfect graphs with perfect order, comparability and permutation graphs. As a consequence, many hard problems become polynomial solvable on GaTEx graphs as well.

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Remarks on the Local Irregularity Conjecture

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A locally irregular graph is a graph in which any two neighboring vertices have distinct degrees. A locally irregular edge coloring of a graph G is an edge coloring such that every color induces a locally irregular subgraph of G and G is colorable if it has such a coloring. Local Irregularity Conjecture claims that every colorable graph requires at most three colors for a locally irregular edge coloring. We consider the conjecture on the class of unicyclic graphs and cacti, where we provide the so called bow-tie graph which contradicts the conjecture.

Relative Brightwell Winkler and shortest path reconfiguration

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The shortest path reconfiguration problem asks, for a pair of shortest paths between two vertices of a graph, whether or not one can find a series of shortest paths between the vertices, which goes from one of the given paths to the other, by changing one vertex at a time.

This problem is in general hard, but is known to be polynomial times solvable, even trivial, on several classes of graphs. We subsume several known show that the problem is trivial on all graphs that admit an NU-polymorphism. We do by proving what we call a relative version of the Brightwell-Winkler characterization of dismantlable graphs H as those for which the Hom-graph $\mathrm{Hom}(G, H)$ is connected for all graphs G.

Partially broken orientations of Eulerian graphs on closed surfaces

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In this talk, we discuss orientations given to edges of Eulerian graphs embedded on closed surfaces, and give a characterization of such a graph having a good orientation, i.e., incoming edges and outgoing edges appear alternately around each vertex of the graph. Furthermore, we consider the partially broken orientations of Eulerian graphs embedded on closed surfaces, that is, one in which only for each of specified vertices, incoming edges and outgoing edges appear alternately around it.

Existence of proper interval vertex coloring

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Proper vertex coloring is called proper closed interval coloring if the set of colors used on the closed neighborhood of every vertex forms an integer interval. In the talk, we present several classes of graphs which admit proper closed interval coloring such as graphs with chromatic numbers at most 3, uniquely colorable graphs, special 4-chromatic graphs, etc. Moreover, we provide multiple constructions of proper closed interval non-colorable graphs; one of such constructions leads to proving the existence of a non-colorable graph of girth at least g for an arbitrary value of $g, g \geq 3$.

Span of a graph

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A recently introduced notion of a graph is used to determine the *maximal safety distance* two players can keep at all times while traversing a graph (the goal: visit either all vertices, or all edges) with accordance to given movement rules. Depending on the goal and the movement rules, different variants of a span of a given connected graph are presented. For each variant, the solution can be obtained by considering only connected subgraphs of a graph product and the projections to the factors. Graphs in which it is impossible to keep a positive safety distance at all moments in time are characterised. It is also shown that the chosen span variant of a given graph can be determined in polynomial time.

Connected Turán number

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As a variant of the well known and extensively investigated Turán numbers, we initiate a systematic study of the maximum number of edges in a graph G on n vertices that does not contain a forbidden F as a subgraph, under the additional constraint that G is connected. The connectivity assumption often causes a drastic change in the extremum.

Despite that the maximum was known for paths since 1977, and the extremal connected graphs for paths were determined in 2008, nothing else was done in this direction for 45 years. We present various general estimates depending on structural parameters of F, and also determine the exact values of the connected Turán numbers of many small forbidden trees.

General position sets in Cartesian product graphs

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For a given graph G, the general position problem asks for the largest set of vertices $S \subseteq V(G)$ such that no three distinct vertices of S belong to a common shortest path of G. The general position problem for Cartesian products of two cycles as well as for hypercubes is considered. The problem is completely solved for the first family of graphs, while for the hypercubes some partial results based on reduction to SAT are given.

Ore's theorem in edge-coloured multigraphs with restrictions in the color transitions

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Let H be a graph possibly with loops and G be a multigraph without loops. An H-coloring of G is a function $c : E(G) \to V(H)$. We will say that G is an H-colored multigraph, whenever we are taking a fixed H-coloring of G.

In this talk, we will introduce the concept of dynamic H-walks and study the existence and length of dynamic H-cycles, dynamic H-trails and dynamic H-paths in H-colored multigraphs. To accomplish this, we will introduce a new concept of color degree, namely, the *dynamic degree*, which allows us to extend some classic results, as the so celebrated Ore's Theorem, for H-colored multigraphs. Also, we will give sufficient conditions for the existence of hamiltonian dynamic H-cycles in H-colored multigraphs, and as a consequence, we obtain sufficient conditions for the existence of properly colored hamiltonian cycle in edge-colored multigraphs, with at least $c \geq 3$ colors.

The distance function on Coxeter like graphs

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Let $S_n(\mathbb{F}_2)$ be the set of all $n \times n$ symmetric matrices with coefficients from the binary field $\mathbb{F}_2 = \{0, 1\}$, and let $SGL_n(\mathbb{F}_2)$ be the subset of all invertible matrices. Let $\tilde{\Gamma}_n$ be the graph with the vertex set $S_n(\mathbb{F}_2)$, where two matrices $A, B \in S_n(\mathbb{F}_2)$ form an edge if and only if $\text{rank}(A - B) = 1$. Let Γ_n be the subgraph in $\tilde{\Gamma}_n$, which is induced by the set $SGL_n(\mathbb{F}_2)$. If $n = 3$, Γ_n is the Coxeter graph. It is well-known that is a distance function on $\tilde{\Gamma}_n$ is given by

$$d(A, B) = \begin{cases} \text{rank}(A - B), & \text{if } A - B \text{ is nonalternate or zero,} \\ \text{rank}(A - B) + 1, & \text{if } A - B \text{ is alternate and nonzero.} \end{cases}$$

Even the Coxeter graph shows that the distance in Γ_n must be different. The main goal is to describe the distance function on this graph.

Edge-connectivity and the number of pairwise disjoint perfect matchings in r-graphs

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For each integer $r \geq 2$, an r-graph is an r-regular graph in which every odd set of vertices is connected by at least r edges to its complement. In the past years, much research about structural properties of r-graphs has been done. In particular, factors and perfect matchings of such graphs were studied; many problems remain unsolved. We focus on the relation between the edge-connectivity and the number of pairwise disjoint perfect matchings in r-graphs.

For $0 \leq \lambda \leq r$ let $m(\lambda, r)$ be the maximum number s such that every λ-edge-connected r-graph has s pairwise disjoint perfect matchings. There are only a few values of $m(\lambda, r)$ known, for instance $m(3,3) = m(4,r) = 1$, and $m(r,r) \leq r-2$ for all $r \neq 5$, and $m(r,r) \leq r-3$ if r is a multiple of 4. In this talk, some upper bounds for $m(\lambda, r)$ will be presented. Furthermore, we discuss relations between the value of $m(5,5)$ and some well-known conjectures for cubic graphs.

This talk is based on a joint work with Yulai Ma, Davide Mattiolo and Eckhard Steffen.

Minisymposium

Algorithmic Graph Theory

Organized by Martin Milanič, *University of Primorska*
Coorganized by Viktor Zamaraev, *University of Liverpool*

INVITED TALK

Structure and algorithms for even-hole-free graphs

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The class of even-hole-free graphs (i.e. graphs that do not contain a chordless cycle of even length as an induced subgraph) has been studied since the 1990's, initially motivated by their structural similarity to perfect graphs. It is known for example that they can be decomposed by star cutsets and 2-joins into algorithmically well understood subclasses, which has led to, for example, their polynomial time recognition. Nevertheless, the complexity of a number of classical computational problems remains open for this class, such as the coloring and stable set problems.

In this talk we survey some of the algorithmic techniques developed in the study of this class.

The tree-independence number of (even hole, diamond, pyramid)-free graphs

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Coauthors: Tara Abrishami, Maria Chudnovsky, Sepehr Hajebi, Sophie Spirkl, Kristina Vušković

In this talk, we will use the central bag method to show that the tree-independence number of (a superclass of) (even hole, diamond, pyramid)-free graphs is bounded. This yields a polynomial time algorithm for the maximum weight independent set problem in the class, and corroborates a special case of a conjecture of Dallard, Milanič, and Štorgel.

Graph drawing and algorithms in modelling of processes and analysis of panel data

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It has recently been observed that presenting ordinal panel data (that records progress of a cohort of subjects S through a series of linearly ordered classes C) results in several interesting graph drawing problems giving rise to newly introduced ordinal panel crossing number. This invariant asks for smallest number of crossings in a graph that results from displaying the progress of subjects through a series of tests T, each of which assigns each subject a category from C. In addition to solving this problem with a polynomial algorithm, we analyze extremal and random instances. We conclude with a question of optimal ordering of categories in C that yields the smallest ordinal panel crossing number of the (modified) input instance and is in this context related (but arbitrarily far from, as our instances show) to feedback arc problem. We conclude with several open problems on using graph algorithms to analyze ordinal panel data.

Isometric Path Complexity of graphs

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Coauthors: Jérémie Chalopin, Florent Foucaud, Yann Vaxès

We introduce and study a new graph parameter, called the *isometric path complexity* of a graph. A path is *isometric* if it is a shortest path between its endpoints. A set S of isometric paths of a graph G is "v-rooted", where v is a vertex of G, if v is one of the end-vertices of all the isometric paths in S. The *isometric path complexity* of a graph G, denoted by $ipco(G)$, is the minimum integer k such that there exists a vertex $v \in V(G)$ satisfying the following property: the vertices of any isometric path P of G can be covered by k many v-rooted isometric paths.

First, we provide an $O(n^2m)$-time algorithm to compute the isometric path complexity of a graph with n vertices and m edges. Then we show that the isometric path complexity remains bounded for graphs in three seemingly unrelated graph classes, namely, *hyperbolic graphs*, *(theta, prism, pyramid)-free graphs*, and *outerstring graphs*. Hyperbolic graphs are extensively studied in *Metric Graph Theory*. The class of (theta, prism, pyramid)-free graphs are extensively studied in *Structural Graph Theory*, *e.g.* in the context of the *Strong Perfect Graph Theorem*. The class of outerstring graphs is studied in *Geometric Graph Theory* and *Computational Geometry*. Our results also show that the distance functions of these (structurally) different graph classes are more similar than previously thought.

Finally, we apply this new concept to the ISOMETRIC PATH COVER problem, whose objective is to cover all vertices of a graph with a minimum number of isometric paths, to all the above graph classes. Indeed, we show that if the isometric path complexity of a graph G is bounded by a constant, then there exists a polynomial-time constant-factor approximation algorithm for ISOMETRIC PATH COVER.

A Parameterized View on $\mathcal{P}$-matchings

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A matching M is a *$\mathcal{P}$-matching* if the subgraph induced by the endpoints of the edges of M satisfies property $\mathcal{P}$. For example, if the property $\mathcal{P}$ is that of being a graph, being a matching, being acyclic, or being disconnected, then we obtain the usual *matching*, an *induced matching*, an *acyclic matching*, and a *disconnected matching*, respectively. First, I will survey the latest developments related to $\mathcal{P}$-matchings from the viewpoint of Parameterized Complexity. Then, I will describe some results focusing majorly on acyclic matching and on three algorithmic paradigms: approximation hardness, kernelization lower bounds, and FPT algorithms with respect to parameters such as *treewidth* and some *below-guarantee parameters*. The second part of the talk is based on the two recent joint works with Meirav Zehavi, appearing in WG'2023.

An algorithm to determine the 3-colorabilty of graphs with minimum degree at least 6

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Proper vertex coloring, the coloring of vertices in which no two adjacent vertices receive the same color, is a category of graph theory that has many applications in computer science and engineering. The fastest known algorithm determining if a general graph is 3-colorable was proved by Beigal and Eppstein in 2013. In this presentation we will explore the subset of graphs in which the minimum degree is bounded and find a new algorithm which improves on Beigel and Eppstein's for this specific class of graphs.

Introduction to tree-independence number

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Tree decompositions are common and useful data structures for designing dynamic programming algorithms for graph problems. In order to obtain efficient algorithms, one looks for a tree decomposition of the input graph that minimizes some measure on the subgraphs induced by the bags. For instance, when this measure is the number of vertices, we obtain the well-known notion of treewidth. In this talk, the considered measure is the independence number. The independence number of a tree decomposition of a graph G is the smallest integer k such that each bag induces a subgraph with independence number at most k. The tree-independence number of G is defined as the minimum independence number over all tree decompositions of G.

We will discuss how to compute tree decompositions with bounded independence number efficiently (when they exist) and their algorithmic utility. We will also mention some classes of graphs that are known to have bounded tree-independence number and discuss the strong relationship between tree-indendence number and the notion of (tw, ω)-boundedness.

Minimizing Dissatisfaction when Allocating Items to Agents with Preference Graphs

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We consider the problem of allocating items to agents, when each item is indivisible and can be assigned to at most one agent. Each agent's preferences over the items are captured in terms of a dedicated directed acyclic graph, the so-called preference graph: its vertices represent items desired by the agent, and an arc (a, b) means that the agent prefers item a over item b. The dissatisfaction of an agent is measured by the number of non-received items which are desired by the agent and for which no more preferred item is allocated to the agent.

We consider the following two problem variants: seeking an allocation of the items to the agents in a way that minimizes (i) the total dissatisfaction over all agents or (ii) the maximum dissatisfaction among the agents. For both problems we study the status of computational complexity involved. We obtain NP-hardness results as well as polynomial algorithms with respect to natural underlying graph structures, such as stars, trees, paths, and matchings.

Besides the general case, we also consider the scenario of identical preferences (in which all agents have the same preference graph), which allows for additional positive results.

Minor-Universal Graph for Graphs on Surfaces

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Coauthor: Cyril Gavoille

We show that, for every n and every surface Σ, there is a graph U embeddable on Σ with at most cn^2 vertices that contains as minor every graph embeddable on Σ with n vertices. The constant c depends polynomially on the Euler genus of Σ. This generalizes a well-known result for planar graphs due to Robertson, Seymour, and Thomas [*Quickly Excluding a Planar Graph.* J. Comb. Theory B, 1994] which states that the square grid on $4n^2$ vertices contains as minor every planar graph with n vertices.

This is a joint work with Cyril Gavoille.

Extending Orthogonal Planar Graph Drawings is Fixed-Parameter Tractable

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Coauthors: Sujoy Bhore, Robert Ganian, Fabrizio Montecchiani, Martin Nöllenburg

The task of finding an extension to a given partial drawing of a graph while adhering to constraints on the representation has been extensively studied in the literature, with well-known results providing efficient algorithms for fundamental representations such as planar and beyond-planar topological drawings. In this paper, we consider the extension problem for bend-minimal orthogonal drawings of planar connected graphs, which is among the most fundamental geometric graph drawing representations. While the problem was known to be NP-hard, it is natural to consider the case where only a small part of the graph is still to be drawn. Here, we establish the fixed-parameter tractability of the problem when parameterized by the size of the missing subgraph. Our algorithm is based on multiple novel ingredients which intertwine geometric and combinatorial arguments. These include the identification of a new graph representation of bend-equivalent regions for vertex placement in the plane, establishing a bound on the treewidth of this auxiliary graph, and a global point-grid that allows us to discretize the possible placement of bends and vertices into locally bounded subgrids for each of the above regions.

Avoidability beyond paths

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The concept of avoidable paths in graphs was introduced in 2019 (Beisegel, Chudnovsky, Gurvich, Milanič, Servatius) as a common generalization of avoidable vertices and simplicial paths. In 2020 (Bonamy, Defrain, Hatzel, Thiebaut) it was proven that every graph containing an induced path of order k also contains an avoidable induced path of the same order. We address this question and specify the concept of avoidability for arbitrary graphs equipped with two terminal vertices. A particularly interesting open problem arising from this can be posed as follows:

Conjecture: Every graph G is either a disjoint union of cliques, or it contains a special induced $P_3 = (s, t, v)$ with the following property: For any selection of $x \in N(s)$ and $y \in N(t)$ such that $G[s, t, v, x, y]$ induces a fork[1], vertices x, y lie in the same component of $G - (N[s, t, v] \setminus \{x, y\})$.

In 2021 (Gurvich, Krnc, Milanič, Vyalyi) showed that the conjecture holds for C_5-free graphs, while it follows from [arXiv:2301.13175] (Chudnovsky, Norin, Seymour, Turcotte) that the conjecture holds for P_5-free graphs. We survey the related positive and negative results, as well as open problems.

Random embeddings of graphs in surfaces

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Coauthor: Jesse Campion Loth

Homeomorphism classes of 2-cell embeddings of a graph in orientable surfaces are in bijective correspondence with rotation systems around each vertex of the graph. If we want to include nonorientable surfaces, we also add a signature $\sigma : E(G) \to \{+1, -1\}$. By taking random local rotations (and a random signature), we can speak about random 2-cell embeddings. In this talk, we present recent proof of the authors that the expected number of faces in a random embedding of any n-vertex graph is $O(n)$. We also discuss its extension to random nonorientable embeddings.

An improvement to the pseudoforest strong nine dragon tree theorem

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A famous theorem of Hakimi says that a graph can be oriented with outdegree at most k if and

[1]Fork graph is a tree obtained from a path P_4, by attaching a pendant vertex to an inner vertex.

only if it's maximum average degree is at most 2k if and only if the edge set can be partitioned into k pseudoforests, that is graphs where each connected component has at most one cycle. Further, one can decide if a graph has maximum average degree at most 2k in polynomial time.

The pseudoforest strong nine dragon tree theorem strengthens Hakimi's theorem in the case when the MAD is not integer. It says every graph with MAD at most $k + d/(k + d + 1)$ decomposes into $k + 1$ pseudoforests where one of the pseudoforests has every component containing at most d edges. Further such a decomposition can be found in polynomial time, and the maximum average degree bound is best possible.

We strengthen this theorem: Every graph with MAD at most $k+d/(k+d+1)$ decomposes into $k+1$ pseudoforests where one of the pseudoforests is : 1) A forest 2) has every component containing at most d edges 3) has every component having diameter at most $2\ell + 2$, where $\ell = \lfloor \frac{d-1}{k+1} \rfloor$, 4) If $d \equiv 1 \mod k+1$, then every component has diameter at most $2\ell + 1$ 5) For all components satisfying $e(K) \geq d - z(k-1) + 1$, for some integer z, then $diam(K) \leq 2z$ for any $z \in \mathbb{N}$.

We provide constructions showing all parameters are best possible, and that such a decomposition can be found in poly time.

Polynomial-Time Approximation Schemes for Independent Packing Problems on Fractionally Tree-Independence-Number-Fragile Graphs

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We investigate a relaxation of the notion of treewidth-fragility, namely tree-independence-number fragility. In particular, we obtain polynomial-time approximation schemes for independent packing problems on fractionally tree-independence-number-fragile graph classes. Our approach unifies and extends several known polynomial-time approximation schemes on seemingly unrelated graph classes, such as classes of intersection graphs of fat objects in a fixed dimension or proper minor-closed classes. We also study the related notion of layered tree-independence number, a relaxation of layered treewidth.

Bisimplicial separators

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A *minimal separator* of a graph G is a set $S \subseteq V(G)$ such that there exist vertices $a, b \in V(G) \setminus S$ with the property that S separates a from b in G, but no proper subset of S does. For an integer $k \geq 0$, we say that a minimal separator is *k-simplicial* if it can be covered by k cliques, and we denote by $\mathcal{G}_k$ the class of all graphs in which each minimal separator is k-simplicial. A 2-simplicial separator is also called *bisimplicial*. Obviously, $\mathcal{G}_0 \subseteq \mathcal{G}_1 \subseteq \mathcal{G}_2 \subseteq \dots$. Classes $\mathcal{G}_0$ and $\mathcal{G}_1$ are well understood: $\mathcal{G}_0$ is the class of all disjoint unions of complete graphs, and by a classical result of Dirac, $\mathcal{G}_1$ is precisely the class of all chordal graphs. In this talk, we present a number of structural and algorithmic results about classes $\mathcal{G}_k$ ($k \geq 2$), with a particular emphasis on $\mathcal{G}_2$.

We show that for each $k \geq 0$, the class $\mathcal{G}_k$ is closed under induced minors, and we also give a complete list of minimal forbidden induced minors for $\mathcal{G}_2$. We further show that, for $k \geq 1$, every nonnull graph in $\mathcal{G}_k$ has a k-simplicial vertex, i.e. a vertex whose neighborhood is the union of k cliques. We also present a decomposition theorem for diamond-free graphs in $\mathcal{G}_2$ (the *diamond* is the graph obtained from the complete graph on four vertices by deleting one edge, and a graph is *diamond-free* if none of its induced subgraphs is isomorphic to the diamond).

Relying on our structural results, as well as results from the literature, we obtain a number of algorithmic consequences, summarized in the table below (as usual, n is the number of vertices and m the number of edges of the input graph).

	diamond-free graphs in $\mathcal{G}_2$	$\mathcal{G}_2$	$\mathcal{G}_k$ ($k \geq 3$)
recognition	$\mathcal{O}(n(n+m))$	?	NP-hard
MAXIMUM WEIGHT CLIQUE	$\mathcal{O}(n(n+m))$	$\mathcal{O}(n^4)$	NP-hard
MAXIMUM WEIGHT STABLE SET	$\mathcal{O}(n^2(n+m))$	$\mathcal{O}(n^6)$	$\mathcal{O}(n^{2k+2})$
VERTEX COLORING	$\mathcal{O}(n(n+m))$	NP-hard	NP-hard

Graph Search Trees and Their Leaves

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Graph searches and their respective search trees are widely used in algorithmic graph theory. The problem whether a given spanning tree can be a graph search tree has been considered for different searches, graph classes and search tree paradigms. Similarly, the question whether a particular vertex can be visited last by some search has been studied extensively in recent years. We combine these two problems by considering the question whether a vertex can be a leaf of a graph search tree. We show that for particular search trees, including DFS trees, this problem is easy if we allow the leaf to be the first vertex of the search ordering. We contrast this result by showing that the problem becomes hard for many searches, including DFS and BFS, if we forbid the leaf to be the first vertex. Additionally, we present structural and algorithmic results for search tree leaves of chordal graphs.

Software for finding and classifying cliques of given size

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Let Γ be a (finite) simple graph and let G be a group of automorphisms of Γ. I will describe hybrid GAP/GRAPE/C software for determining G-orbit representatives for the cliques of given size (or for the maximal cliques of given size) in Γ. This software is designed to exploit the graph symmetries provided by G and may be used for parallel computation on an HPC cluster, such as the QMUL Apocrita cluster. A research application will be presented.

A New Temporal Interpretation of Cluster Editing

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Coauthors: Cristiano Bocci, Chiara Capresi, Kitty Meeks

The NP-complete graph problem CLUSTER EDITING seeks to transform a static graph into a disjoint union of cliques by making the fewest possible edits to the edges. We introduce a natural interpretation of this problem in temporal graphs, whose edge sets change over time. This problem is NP-complete even when restricted to temporal graphs whose underlying graph is a path, but we obtain two polynomial-time algorithms for restricted cases. In the static setting, it is well-known that a graph is a disjoint union of cliques if and only if it contains no induced copy of P_3; we demonstrate that no general characterisation involving sets of at most four vertices can exist in the temporal setting, but obtain a complete characterisation involving forbidden configurations on at most five vertices. This characterisation gives rise to an FPT algorithm parameterised simultaneously by the permitted number of modifications and the lifetime of the temporal graph.

Universal obstructions in graph minors

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A parameterized graph is a sequence $H_1, H_2, ...$ such that H_i is a minor of H_{i+1} for all i. Minor monotone classes of bounded treewidth can be described asymptotically by the exclusion of a single parameterized graph: The grid. A natural question is to ask if a similar theorem exists for every minor-monotone graph parameter. More precisely; Is every minor-closed parameter asymptotically equivalent to the exclusion of a finite set of parameterized graphs? In this talk I present an overview of recent results and techniques that lead to a lattice of minor-monotone graph parameters in the style of Robertson's and Seymour's graph minor series, with the Hadwiger number as a global maximum. This is joint work with Christophe Paul, Evangelos Protopapas , and Dimitrios Thilikos.

Vertex-decomposable graphs and the INDEPENDENT SET problem

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Vertex-decomposability is a tool that has been useful in the topological combinatorics and combinatorial commutative algebra literatures. We overview the connection between vertex-decomposable graphs and the algorithmic problems of finding a maximum independent set in a graph, or more generally of extending a given independent set to a largest possibly maximal one. In particular, we will show that both problems have efficient solutions among the class of graphs whose only holes have length 5.

Lollipop and Cubic Weight Functions for Graph Pebbling

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Given a distribution of pebbles on the vertices of a graph G, a pebbling move removes two pebbles from a vertex and puts one pebble on an adjacent vertex. The pebbling number of a graph G is the smallest number of pebbles required such that, given an arbitrary initial configuration of pebbles, one pebble can be moved to any vertex of G through some sequence of pebbling moves. We improve the weight function technique, introduced by Hurlbert and extended by Cranston et al., that gives an upper bound for the pebbling number of graphs by constructing a non-tree weight function for Q_4. Then, we propose a conjecture on weight functions for the n-dimensional cube. We also construct a set of valid weight functions for variations of lollipop graphs, extending previously known constructions.

Minisymposium

Applications Of Graphs In Algebra, Linear Algebra, And Functional Analysis

Organized by Bojan Kuzma, *University of Primorska*
Coorganized by Damjana Kokol Bukovšek, *University of Ljubljana*

INVITED TALK

The synchronisation hierarchy for permutation groups

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The concept of a synchronising permutation group was introduced over 15 years ago as a possible way of approaching The Černý Conjecture from automata theory. Such groups must be primitive. In an attempt to understand synchronising groups, a whole hierarchy of properties for a permutation group has been developed, namely, 2-transitive groups, $\mathbb{Q}$I-groups, spreading, separating, synchronsing, and primitive. Many surprising connections with other areas of mathematics such as finite geometry, graph theory, and design theory have arisen in the study of these properties. In this talk I will discuss some of the connections with graph theory and some recent results about where groups sit in the hierarchy.

INVITED TALK

On characterization of a finite group by its Gruenberg-Kegel graph

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The Gruenberg-Kegel graph (or the prime graph) of a finite group G is the simple graph whose vertices are the prime divisors of $|G|$, with primes p and q adjacent in this graph if and only if G contains an element of order pq. In 2022, P.J. Cameron and the speaker proved that if there are only finitely many groups whose Gruenberg-Kegel graphs coinside with the Gruenberg-Kegel graph of a group G, then G is forced to be almost simple, in particular, any group which is uniquely determided by its Gruenberg-Kegel graph is almost simple. In this talk we discuss a resent progress in characterization of finite simple groups by their Gruenberg-Kegel graphs.

This talk is based on a series of joint works with P.J. Cameron, W. Guo, K.A. Ilenko, A.P. Khramova, A.S. Kondrat'ev, L.G. Nechitailo, V.V. Panshin, and A.M. Staroletov.

The distant graph of the projective line over a finite ring with unity

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We discuss the projective line $\mathbb{P}(R)$ over a finite associative ring with unity. It is defined as the set of free cyclic submodules $R(a, b)$ generated by admissible pairs (a, b) . $\mathbb{P}(R)$ is naturally endowed with the symmetric and anti-reflexive relation "distant". We study the graph of this relation on $\mathbb{P}(R)$ and classify up to isomorphism all distant graphs $G(R, \Delta)$ for rings R up to order p^5, p prime. It is demonstrated that the description of projective lines over rings of prime power order suffices to characterize the projective line over finite ring.

Blocking sets and minimal codes from expander graphs

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Coauthors: Noga Alon, Shagnik Das, Alessandro Neri

A strong blocking set is a collection of one-dimensional subspaces of a finite vector space that meets every hyperplane in a spanning set. Constructing small strong blocking sets is equivalent to constructing short minimal linear codes. We will present an explicit construction of strong blocking sets that are only a constant factor away from the lower bounds, thus resolving one of the main open problems on minimal linear codes. Our construction combines explicit expander graphs with asymptotically good error-correcting codes, using a geometric argument.

On compressed commuting graphs

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Coauthors: Damjana Kokol Bukovšek, Nik Stopar

In this talk it will be presented results of cooperation between group of researchers from Bosnia and Herzegovina and group of researchers from Slovenia. We examined standard commuting graph and his compression. Examples are provided to illustrate the compressed commuting graphs of finite fields and rings of 2x2 matrices over small fields.

Metric dimensions of graphs arising from rings and semirings

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The metric dimension of a graph is the minimum cardinality of a subset S of vertices such that all other vertices are uniquely determined by their distances to the vertices in S. Finding the metric dimension of a graph is an NP-hard problem. We survey recent results regarding metric dimensions of some graphs that arise from rings and semirings.

Applications of plane trees to matrix integrability

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The talk will be based on the joint work with Suren Danielyan, Elena Kreines, and Fedor Pakovich.

The notion of matrix integration was introduced in 2007 by Bhat and Mukherjee, see [1], as a natural counterpart to the classical notion of matrix differentiation. It has several motivations in the geometry of polynomials and related areas. Not every matrix has an integral, and the problem of the existence of integrable and non-integrable matrices with a given Jordan structure remained open since [1]. We present a complete solution of this problem. Although we established an easy to check new criteria for matrix integrability.

References:
[1] B.V.R. Bhat, M. Mukherjee, Integrators of matrices, Linear Algebra Appl. 426 (2007) 71-82.

Trifactorization of pattern symmetric nonnegative matrices

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Coauthor: Helena Šmigoc

Let A be symmetric entrywise nonnegative matrix of order $n \times n$. A factorization $A = BCB^T$, where B is nonnegative matrix of order $n \times k$ and C is symmetric nonnegative matrix of order $k \times k$, is called *symmetric nonnegative trifactorization* of A. Minimal possible k in such factorization is called the *SNT-rank* of A. In the talk we will for the most part take aside the actual values of matrices and only consider their patterns. Our main focus will be the question, how small can SNT-rank be among all symmetric nonnegative matrices A with given zero-nonzero pattern. The pattern of a matrix can be described by a simple graph that allows loops. We will answer this question for trees and complete graphs without loops.

Graph of the relation induced by Birkhoff-James orthogonality.

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Coauthors: Ljiljana Arambašić, Alexander Guterman, Rajna Rajić, Svetlana Zhilina

The relation of orthogonality can be generalized in various ways from Euclidean to general normed spaces. One of the better known generalizations is the Birkhoff-James orthogonality, which is defined in one of its equivalent forms as $x \perp y$ if y lies in the kernel of the support functional for x. This relation is homogeneous in both factors, but unlike in the Euclidean space, it is not necessarily additive or symmetric. We assign a (directed) graph to this relation, where non-zero vectors are vertices and two orthogonal vectors are connected. We will show some properties of this graph and the graph of a related relation introduced over C^*-modules, and their use in studying homomorphisms of the relation (i.e. not necessarily linear maps that preserve orthogonality). We will also show that this relation can be used to calculate the dimension of the space and determine if the norm is smooth or strictly convex, and that Birkhoff-James orthogonality completely classifies the norm up to (conjugate) linear bijection. It should be noted that in the case of infinite-dimensional spaces, A. Blanco and A. Turnsek had already reached similar result in 2006.

These results are a joint work of Lj. Arambašić, A. Guterman, R. Rajić, and S. Zhilina.

Graphs of linear codes

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Linear $[n, k]_q$ codes are k-dimensional subspaces of an n-dimensional vector space over the field with q-elements. Non-degenerate codes are codes whose generating matrices contain no zero columns. Projective codes are codes whose every two columns of their generating matrices are non-proportional. In general codes whose every t columns of their generating matrices are linearly independent are dual codes with minimal Hamming distance at most $t + 1$.. The projective codes of maximal length are simplex codes.

Two distinct linear codes of any of those classes are adjacent vertices of the Grassmann graph if they have the maximal possible number of common codewords, i.e. the dimension of their intersection is $k - 1$ dimensional. They induce subgraphs of the Grassman graphs

We present overview of results regarding those subgraphs, including some general properties of those classes and a specific example of the graph of simplex $[5, 2]_4$ codes.

Sparsity and Regularity of Graphs with Two Distinct Eigenvalues

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For a connected graph G on n vertices, let $\mathcal{S}(G)$ be the set of all real symmetric $n \times n$ matrices $A = [a_{ij}]$ such that $a_{ij} = 0$ if and only if $\{i, j\}$ is not an edge of G. In this talk, we present some recent results about graphs with minimum edge density and two distinct eigenvalues. Using similar techniques, we give a characterization of connected $r-$regular graphs G with $r \leq 4$ for which there is a matrix in $\mathcal{S}(G)$ with two distinct eigenvalues. This is joint work with AIM-q group.

The strong spectral properties for graphs

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The inverse eigenvalue problem of a graph G is the problem of characterising all lists of eigenvalues of real symmetric matrices whose off-diagonal pattern is prescribed by the adjacencies of G. The strong spectral property is a powerful tool in this problem that identifies matrices whose entries can be perturbed while controlling the pattern and preserving the eigenvalues.

We will present the strong spectral property and the liberation set of a graph G, which is independent of the choice of a matrix corresponding to G. Moreover, we will investigate a surprising connection between the zero forcing game on Cartesian products of graphs and the liberation set of their union.

Some open problems on vertex-transitive/regular complementary prisms and self-complementary graphs

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Let Γ be a finite simple graph on n vertices. The *complementary prism* $\Gamma\bar{\Gamma}$ is obtained from the disjoint union of Γ and its complement $\bar{\Gamma}$ if we add an edge between each pair of identical vertices in Γ and $\bar{\Gamma}$. It is known that $\Gamma\bar{\Gamma}$ is vertex-transitive if and only if Γ is vertex-transitive and self-complementary. In the talk I will discuss some open problem about vertex-transitive/regular complementary prisms and self-complementary graphs that involve:

- graph spectrum,
- hamiltonian properties,
- graph homomorphisms.

Point-line geometries related to binary equidistant codes

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We investigate point-line geometries whose singular subspaces correspond to binary equidistant codes. The main result is a description of automorphisms of these geometries. Generally, such automorphisms are induced by coordinate permutations (monomial linear automorphisms). However, in some important cases (for example, in the case of simplex codes), automorphisms induced by non-monomial linear automorphisms surprisingly arise.

Some results in the spirit of Chow's theorem

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Classical Chow's theorem concerns Grassmann spaces, but it can also be presented in a graph-theoretic approach. The Grassmann graph $\Gamma_m(V)$ is the graph whose vertices are m-dimensional subspaces of a vector space V, and two subspaces are adjacent (connected by an edge) if their intersection is $(m-1)$-dimensional. In this setting Chow's theorem describes automorphisms of $\Gamma_m(V)$.

There are many results resembling Chow's theorem in other geometries. In this talk I briefly discuss some of them: in spine spaces, in non-degenerate linear codes, in spaces of matrices, and especially in Hilbert Grassmannians.

Projections in a Hilbert space can be characterized as self-adjoint idempotents in the algebra of bounded operators. Two projections (of the same rank) are adjacent if their images are adjacent subspaces. Thus, Chow's theorem for Hilbert Grassmannians can be easily translated into projections. Here a natural question arises: can we obtain similar results for some wider class of oparators? We consider the graph $\Gamma_{\mathcal{C}}$, whose vertex set is a conjugacy class $\mathcal{C}$ consisting of finite-rank self-adjoint operators on a complex Hilbert space H. The case when the operators from $\mathcal{C}$ have two eigenvalues only, is covered by Chow's theorem. Under the assumption that operators from $\mathcal{C}$ have more than two eigenvalues we show that every automorphism of the graph $\Gamma_{\mathcal{C}}$ is induced by a unitary or anti-unitary operator up to a permutation of eigenspaces with the same dimensions.

On extremal ternary self-dual codes of length 36

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It is known that the Pless symmetry code $C(q)$ of length $n = 2q + 2$, where $q \equiv -1 \pmod 3$ is an odd prime power, contains a set of n codewords of weight n, which after replacing every entry equal to 2 with -1 form the rows of a Hadamard matrix equivalent to the Paley-Hadamard matrix of type II. In particular, the Pless symmetry code $C(17)$ contains the rows of a Hadamard matrix P of Paley type II, having a full automorphism group of order $4 \cdot 17(17^2 - 1) = 19584$, and the rows of P span the code $C(17)$.

In this talk, we report the existence of a regular Hadamard matrix H^* which is monomially equivalent to the Paley-Hadamard matrix of type II such that the symmetric 2-$(36, 15, 6)$ design associated with H^* has a full automorphism group of order 24 and its (0,1)-incidence matrix spans a code equivalent to $C(17)$. Furthermore, we classified all symmetric 2-$(36, 15, 6)$ designs that admit an automorphism of order 2 and their incidence matrices span an extremal ternary self-dual code of length 36. The results of this classification imply the following.

Theorem 1. *(a) Up to isomorphism, there exists exactly one symmetric 2-$(36, 15, 6)$ design D that admits an automorphism of order 2 and its incidence matrix spans an extremal ternary self-dual code of length 36.*

(b) The full automorphism group G of D is of order 24, and G is isomorphic to the symmetric group S_4.

(c) The regular Hadamard matrix associated with D is equivalent to the Paley-Hadamard matrix of type II.

(d) The ternary code spanned by the incidence matrix of D is equivalent to the Pless symmetry code.

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Compressed zero-divisor graphs of rings

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A zero-divisor graph of a ring R is a graph whose vertices are nonzero zero-divisors of R and there is an edge from x to y if and only if $x \neq y$ and $xy = 0$. The idea of compression is to replace sets of vertices with equal neighborhoods by a single vertex in order to make the graph smaller. In this talk we present a specific type of compression that allows us to extended the formation of a compressed zero-divisor graph to a functor from the category of rings to the category of directed graphs. This means that our version of compressed zero-divisor graph takes into account not only the zero-divisor structure of the underlying ring, but also the structure of its homomorphic images. We focus on the interplay between the combinatorial properties of a compressed zero divisor-graph and the algebraic properties of the underlying ring, and present some examples of properties that can be characterized using graphs. We discuss in more details the compressed zero-divisor graph of a matrix ring $M_n(F)$ over a finite field F, show that it can be characterized by a set of axioms, and discuss the isomorphism problem for this graph. The results we present imply that the multiplicative and the additive structure of the ring $M_n(F)$ are uniquely determined by its zero-divisor structure.

Spectral arbitrariness for trees fails spectacularly

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The Inverse Eigenvalue Problem for a Graph (IEPG) asks for all possible spectra of real symmetric matrices whose pattern of off-diagonal nonzero entries is specified by a given graph. A subproblem to the IEPG is to determine all possible ordered multiplicity lists of eigenvalues. Once an ordered multiplicity list m is known to be realisable for a graph G, one needs to determine the set of all possible assignments of eigenvalues for m. If every set of distinct eigenvalues can be assigned to m, then we say that m is spectrally arbitrary for G. We present an infinite family of trees and ordered multiplicity lists whose sets of realising eigenvalues are highly constrained. Furthermore, we exhibit examples where multiplicity lists can only be achieved by a unique (up to shifting and scaling) set of eigenvalues.

Proving algebraic theorems using spectral graph theory

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In this talk, we discuss how to use spectral graph theory in a general way to find or count subgraphs in graphs. Using this general framework, one may prove theorems in combinatorial number theory or discrete geometry about objects like sumsets, “distances" in a vector space over a finite field, or sets of vectors with prescribed dot-product.

Zigzags in tetrahedral chains and the associated Markov chain

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Zigzags are important objects considered in graphs embedded in surfaces and complexes. We investigate zigzags in tetrahedral chains and construct the associated Markov chain whose states are z-monodromies. Using this construction we find the probability for each possible number of zigzags in tetrahedral chains.

Compatibility in Grassmann graphs

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Compatibility of subspaces translates into commutativity of projections. Together with well known adjacency it gives rise to ortho-adjacency. The latter concept is not new but we give it a new application in Grassmann graphs over Hilbert spaces.

In a complex Hilbert space H we deal with the ortho-Grassmann graph $\Gamma_k^{\perp}(H)$ whose vertices are k-dimensional subspaces of H and two such subspaces are connected by an edge when they are ortho-adjacent, meaning that they are compatible and adjacent. We prove that every automorphism of the graph $\Gamma_k^{\perp}(H)$ is induced by a unitary or anti-unitary operator, except the case $\dim H = 2k$. If $\dim H = 2k \geq 6$, then additionally, compositions of such automorphisms and the orthocomplementary map are possible, while for $\dim H = 2k = 4$ the statement fails.

Applying characterization of adjacency in terms of ortho-adjacency and an analogue of Chow's theorem for conjugacy classes of finite-rank self-adjoint operators, we extend our result on generalised ortho-Grassmann graphs associated to such conjugacy classes.

The result has been achieved together with Mark Pankov and Krzysztof Petelczyc.

Minisymposium

Association Schemes And Related Algebras

Organized by Jae-Ho Lee, *University of North Florida*
Coorganized by
Paul Terwilliger, *University of Wisconsin - Madison*
Štefko Miklavič, *University of Primorska*

INVITED TALK

Graph with three eigenvalues and the coherent closure

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Graphs with three distinct eigenvalues are fundamental objects of study in spectral graph theory. The most well-known examples are strongly regular graphs. In 1995, Willem Haemers posed a question at the 15th British Combinatorial Conference: "Do there exist any connected graphs having three distinct eigenvalues apart from strongly regular graphs and complete bipartite graphs?" This question prompted responses from Muzychuk-Klin and Van Dam, who found new families of non-regular graphs with three distinct eigenvalues.

Muzychuk and Klin initiated the study of a graph with three distinct eigenvalues via its Weisfeiler-Leman closure (also known as the coherent closure). They classified such graphs whose Weisfeiler-Leman closure has rank at most 7. In this talk, I will provide a brief overview of the history of non-regular graphs with three distinct eigenvalues, as well as present our recent results on such graphs whose Weisfeiler-Leman closure has a small rank. Our results include a correction of the literature (where the rank 8 case was erroneously claimed to be impossible) and the discovery of a new biregular graph with three distinct eigenvalues obtained from a quasi-symmetric design.

This talk is based on joint work with Jose Yip.

INVITED TALK

Bounding the Fractional Chromatic Number with Eigenvalues Using Symmetry

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Given a graph G, we let $s^+(G)$ denote the sum of the squares of the positive eigenvalues of the adjacency matrix of G, and we similarly define $s^-(G)$. We prove that

$$\chi_f(G) \geq 1 + \max\left\{\frac{s^+(G)}{s^-(G)}, \frac{s^-(G)}{s^+(G)}\right\}$$

and thus strengthen a result of Ando and Lin, who showed the same lower bound for the chromatic number $\chi(G)$. Using ideas motivated by symmetry reduction and association schemes, we show a stronger result wherein we give a bound using the eigenvalues of G and H whenever G has a homomorphism to an edge-transitive graph H.

Association schemes on anisotropic points

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The content of this talk is based on *polar spaces*. These are certain incidence geometries, classically embedded in a projective space. The points in the ambient projective space outside of the embedded polar space are called *anisotropic*. The geometry of a polar space embedded in a finite projective space often gives rise in a natural way to an association scheme on the anisotropic points with few classes. Most of the cases have been studied, see e.g. [2, §3.1] for a nice summary. In this talk we discuss the missing cases.

Recently polarity graphs on anisotropic points have been used to give good constructions of clique-free pseudorandom graphs [1]. Using the association schemes discussed above, we can explicitly compute the eigenvalues of some polarity graphs on anisotropic points.

References:

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Spectra of normal Cayley graphs

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A *normal Cayley graph* for a group G is a Cayley graph whose connection set is a union of conjugacy classes of G. Equivalently, it is a graph that lies in the conjugacy class scheme of a group, so we can use tools from association schemes to study its spectrum. A graph is said to be *integral* if it has only integer eigenvalues. In this talk, we will show that an integral normal Cayley graph for a group of odd order has an odd eigenvalue.

Some open questions on distance-regular graphs with primitive automorphism groups

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In recent years, I have been working with students on cataloguing distance-regular graphs with primitive automorphism groups, using the libraries of primitive groups in the GAP computer algebra system. In this talk, I will discuss some of the more curious observations we made, and present some open questions that have arisen from this work. These in turn relate to topics such as Cayley graphs, association schemes and finite geometries, as well as permutation groups.

Multivariate P- and/or Q-polynomial association schemes

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Coauthors: Hirotake Kurihara, Da Zhao, Yan Zhu

The details are available in the arXiv paper by these four authors : Multivariate P- and/or Q-

polynomial association schemes (arXiv: 2305.00707).

The classification problem of P- and Q-polynomial association schemes has been one of the central problems in algebraic combinatorics. Generalizing the concept of P- and Q-polynomial association schemes to multivariate cases, namely to consider higher rank P- and Q-polynomial association schemes, has been tried by some authors, but it seems that so far there were neither very well-established definitions nor results. Very recently, Bernard, Crampé, d'Andecy, Vinet, and Zaimi : Bivariate P-polynomial association schemes (arXiv:2212.10824), defined bivariate P-polynomial association schemes, as well as bivariate Q-polynomial association schemes. Inspired by their paper, we study these concepts and propose a new modified definition concerning a general monomial order, which is more general and more natural and also easy to handle. We prove that there are many interesting families of examples of multivariate P- and/or Q-polynomial association schemes.

Classification of the tight Euclidean 5-designs in $\mathbb{R}^2$

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We first recall the definitions of Euclidean t-designs and tight Euclidean t-designs. The classifications of tight Euclidean t-designs in the Euclidean space $\mathbb{R}^n$ are interesting and difficult problems. For the case $t = 2$ and $t = 3$, the classifications of tight t-designs were essentially completed by Bannai-Bannai-Suprijanto (2007) and by Et.Bannai (2006) respectively. For $t \geq 4$ Euclidean tight t-designs on two concentric spheres in $\mathbb{R}^n$ have very much studied by many authors including ourselves, although the complete classification is not obtained yet. The cases when the numbers of the spheres are more than 2 is not studied so much, so far. In this talk we consider the Euclidean tight 5-design in $\mathbb{R}^2$ with no restriction on the number of concentric spheres, as the smallest open case. In this case the size of the tight design is only 8 and the number of spheres (circles) is at most 4. We give a complete classification and the parametrization of these tight designs. There exist a lots of such examples with many freedoms than we originally expected.

Doubly Almost-bipartite Leonard Pairs

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In this talk, we recall the notion of a Leonard pair A,A^* on a vector space V of dimension $d+1$. We say that the pair A,A^* is **doubly almost-bipartite** (DAB) whenever the tridiagonal matrix representing $A \in \mathrm{Mat}_{d+1}(\mathbb{K})$ satisfies

$$A_{i,i} = 0 \quad \text{if and only if} \quad 1 \leq i \leq d-1.$$

In particular, both $A_{0,0}$ and $A_{d,d}$ are assumed to be non-zero. Using this definition, we classify (up to isomorphism) the doubly almost-bipartite Leonard pairs. The classification reveals that these Leonard pairs are of the q-Racah, q-Krawtchouk, or q-Hahn type. A number of examples will be presented, along with some connections to distance-regular graphs and the irreducible modules for the Terwilliger algebras of certain association schemes.

A graph co-spectral to $\mathrm{NO}^{+}(8,2)$

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Coauthors: Sam Adriaensen, Robert Bailey, Morgan Rodgers

The graph $\mathrm{NO}^{+}(8,2)$ is strongly regular with parameters $(120,63,30,36)$. It can be constructed using a quadratic form of Witt index 4 on $\mathrm{GF}(2)^8$. Then its vertices are the set of non-singular vectors. Two vertices are adjacent if and only if they are orthogonal with relation to the quadratic form. Its automorphism group is $\mathrm{P}\Gamma\mathrm{O}^{+}(8,2)$.

In their recent book – Strongly Regular Graphs – Brouwer and Van Maldeghem mention the existence of a non-isomorphic, strongly regular graph with the same parameters, admitting $\mathrm{Sym}(7)$ as automorphism group. In this talk we discuss how the adjacency relation of $\mathrm{NO}^{+}(8,2)$ can be modified to obtain this graph, it turns out that the unique ovoid (and spread) of the triality quadric $\mathrm{Q}^{+}(7,2)$ plays a central role. We also discuss further interesting properties such as that fact the cliques and co-cliques get switched by modifying the adjacency relation of $\mathrm{NO}^{+}(8,2)$.

Erdős-Ko-Rado theorems for finite general linear groups

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We call a subset Y of the finite general linear group $\mathrm{GL}(n,q)$ *t-intersecting* if $\mathrm{rk}(x-y)\le n-t$ for all $x,y\in Y$. In this talk we give upper bounds on the size of t-intersecting sets and characterise the extremal cases that attain the bound. This is a q-analog of the corresponding result for the symmetric group, which was conjectured by Deza and Frankl in 1977 and proved by Ellis, Friedgut, and Pilpel in 2011. The results are obtained by using eigenvalue techniques and the theory of association schemes plays a crucial role.

This is a joint work with Kai-Uwe Schmidt.

Terwilliger algebras beyond local distance-regularity: A combinatorial approach

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The Terwilliger algebra T has been extensively studied in the context of distance-regular graphs, which have only a few irreducible T-modules (up to isomorphism) of a specific endpoint, all of which are (non-)thin. These investigations seek to establish algebraic conditions that hold if and only if certain combinatorial conditions are met. This talk aims to extend these results to irreducible T-modules with endpoint 0 and with endpoint 1 of certain (not necessarily distance-regular) graphs, and shed light on their combinatorial properties.

Let Γ be a finite, simple, and connected graph, and let x be a vertex of Γ that is not a leaf. We examine which vertices x of Γ admit a Terwilliger algebra $T=T(x)$ with a unique irreducible T-module with endpoint 0 and exactly one irreducible T-module (up to isomorphism) with endpoint 1, which are both thin. We give a purely combinatorial characterization of these algebraic properties, which involves the number of some walks in Γ of a specific shape.

Set-theoretic hypergroups

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Recent studies of hypergroups have led to a consideration of "set-theoretic hypergroups". Given a nonempty set X, and two subsets $p, q \subset X \times X$, we define $p \circ q$ to be the set of $(x, z) \in X \times X$ such that for some $y \in X$, $(x, y) \in p$ and $(y, z) \in q$. A set-theoretic hypergroup consists of a partition P of $X \times X$, containing 1_X, closed under transposition, and such that for any three elements $p, q, r \in P$, $r \cap (p \circ q) \neq \emptyset$ implies $r \subseteq p \circ q$. Each set-theoretic hypergroup naturally determines a hypergroup. This talk will present some recent results on the existence of set-theoretic hypergroups determining the various non-symmetric hypergroups with four elements.

The q-Onsager algebra and the quantum torus

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The q-Onsager algebra, denoted O_q, is encountered in the study of distance-regular graphs and association schemes. This algebra has two generators, W_0 and W_1. The structure of this algebra is similar to that of $U_q(sl_2)$, and there is a series of elements of O_q, called alternating elements, that are defined recursively. However, what they look like as polynomials in these generators remains an open problem.

In this talk, we display an algebra T_q (called the *quantum torus*) and describe the images of the alternating elements under a homomorphism p from O_q to T_q. These images can be expressed in closed form in a specific basis, as well as in matrix form and in terms of generating functions.

Quotient-polynomial Coxeter graphs

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A quotient-polynomial graph (defined by Fiol in 2016) is a graph whose adjacency matrix generates the adjacency algebra of a symmetric association scheme, one of whose relations is the graph. Distance-regular graphs and cyclotomic graphs provide examples of quotient-polynomial graphs, but there are many others. We will begin this talk with the problem of determining an adequate style for describing the parameters of general quotient polynomial graphs - along the lines of what intersection arrays provide for distance-regular graphs; and explore feasibility conditions available for these parameters.

We will then consider Coxeter graphs (defined for finite Coxeter groups in BCN) as a source of quotient-polynomial graphs that are not always distance-regular. We will explore the quotient-polynomial graph parameters for the non-distance-regular Coxeter graphs of type $B_{n,k}$. We will then consider finite symmetric association schemes arising from maximal parabolic Hecke algebras of affine types $\tilde{C}_{n,0}$, $\tilde{F}_{n,0}$, and $\tilde{G}_{2,0}$, and see specific families of these that come from the finite abstract regular polytopes of these types. This talk based on a merging of joint work with my former Ph.D. students Alyeah Alsairafi, Roqayia Shalabi, and Roghayeh Maleki.

The Clebsch-Gordan coefficients of U(sl2) and the Terwilliger algebras of Johnson graphs

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In 2018, I published a paper entitled “An algebra behind the Clebsch–Gordan coefficients of Uq(sl2)”. In this talk, I would like to talk about the connection between the Terwilliger algebras of Johnson graphs and the algebra behind the Clebsch-Gordan coefficients of U(sl2).

Distance-regular graphs with tails

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Let Γ be a distance-regular graph with valency $k \geq 3$ and diameter $d \geq 2$. It is well-known that the entry-wise product $E \circ F$ of any two minimal idempotents of Γ is a linear combination of minimal idempotents of Γ. In the case when $E = F$, the rank one minimal idempotent E_0 is always present in this linear combination and can be the only one only if $E = E_0$ or $E = E_d$ and Γ is bipartite. We study the case when $E \circ E \in \mathrm{span}\{E_0, E, H\}$ for some minimal idempotent H of Γ. We call a minimal idempotent E with this property a *tail*. If Γ is Q-polynomial with respect to a minimal idempotent E, then E is a tail. Let θ be an eigenvalue of Γ not equal to $\pm k$ and with multiplicity m. We show that

$$mb_1^* \, (b_1 + k\omega + 1)\,(k\omega + 1) \;\geq\; kb_1\,(b_1^* + m\omega + 1)\,(m\omega + 1),$$

where $\omega = \theta/k$, $a_1^* = q_{ii}^i$ for $\theta = \theta_i$ and $b_1^* = m - 1 - a_1^*$. Let E be the minimal idempotent corresponding to θ. The equality case is equivalent to E being a tail. Further characterizations of the case when E is a tail are given.

This is joint work with Paul Terwilliger.

Computing association schemes and their properties in GAP

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University of Canterbury, New Zealand

The ability to work with association schemes computationally can be an incredible asset to research. It allows one to find examples or counter examples, observe patterns and form conjectures, or simply “get a feel” for a particular scheme.

In this talk I will introduce my GAP package with J. Bamberg and A. Hanaki, *AssociationSchemes*. I will provide an overview of its implementation and give a demonstration of its functionality. This may include: constructions of common association schemes, Schurian schemes, computing automorphism groups, determining isomorphism of schemes, finding fusion schemes, computing character tables, computing simultaneous eigenspaces, determining Delsarte designs, and finding P and Q-polynomial orderings.

Some recent applications of the software to research will also be discussed.

Distance-Biregular Graphs and Coherent Configurations

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Distance-biregular graphs were proposed by Delorme, and extend the class of distance-regular graphs. The same way that the adjacency algebra of a distance-regular graph gives rise to an association scheme, a partitioning of the distance adjacency matrices gives rise to a particular kind of coherent configuration. The halved graphs of a distance-biregular graph are both distance-regular, so the underlying coherent configuration is related to two assocation schemes. In this talk, we will discuss some results on distance-biregular graphs and their coherent configurations.

Circular Hessenberg pairs

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In this talk, we introduce a linear algebraic object called a circular Hessenberg pair. A square matrix is called *Hessenberg* whenever each entry below the subdiagonal is zero and each entry on the subdiagonal is nonzero. Let M denote a Hessenberg matrix. Then M is called *circular* whenever the upper-right corner entry of M is nonzero and every other entry above the superdiagonal is zero. A *circular Hessenberg pair* consists of two diagonalizable linear maps on a nonzero finite-dimensional vector space, each acting on an eigenbasis of the other one in a circular Hessenberg fashion. Let A, A^* denote a circular Hessenberg pair. We investigate six bases for the underlying vector space that we find attractive. We display the transition matrices between certain pairs of bases among the six. We also display the matrices that represent A and A^* with respect to the six bases. We introduce a special type of circular Hessenberg pair, said to be *recurrent*. We show that a circular Hessenberg pair A, A^* is recurrent if and only if A, A^* satisfy the tridiagonal relations. For a circular Hessenberg pair, there is a related object called a *circular Hessenberg system*. We classify up to isomorphism the recurrent circular Hessenberg systems. To this end, we construct four families of recurrent circular Hessenberg systems. We show that every recurrent circular Hessenberg system is isomorphic to a member of one of the four families.

Distance-regular graphs with classical parameters which support a uniform structure

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Let G be a connected bipartite graph. Then, its adjacency matrix A can be decomposed as $A = L + R$, where $L = L(x)$ and $R = R(x)$ are respectively the lowering and the raising matrices, with respect to a certain vertex x. The graph G has a *uniform structure* with respect to x if the matrices RL^2, LRL, L^2R, and L satisfy a certain linear dependency.

Let $\Gamma = (X, E)$ be a connected non-bipartite graph. Fix a vertex $x \in X$ and let $\Gamma_f = (X, E_f)$ be the bipartite graph, where $E_f = E \setminus \{yz \mid \partial(x, y) = \partial(x, z)\}$ and ∂ is the distance function in Γ. The graph Γ is said to support a uniform structure whenever Γ_f has a uniform structure with respect to x.

Assume that Γ is a non-bipartite distance-regular graph with classical parameters (D, q, α, β). It turns out that q is an integer different from 0 and -1. In this talk, I will present the complete classification of non-bipartite distance-regular graphs with classical parameters (D, q, α, β), for $q \leq 1$ and $D \geq 4$, that support a uniform structure.

Quantum isomorphic graphs from association schemes

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Quantum games have emerged as useful tools to understand the power of shared entanglement. A simple classical game can be used to define graph isomorphism: two players, Alice and Bob, convince a referee that graphs G and H are isomorphic as follows. Alice and Bob may strategize beforehand but cannot communicate during the game. The referee gives Alice (Bob) a vertex x_A (x_B) in $V(G)\dot{\cup}V(H)$. Alice and Bob respond with vertices y_A, y_B of the opposite graph and win if the relationship (equal, adjacent, non-adjacent) between the two vertices of G matches the relationship between the two vertices (among x_A, y_A, x_B, y_B) belonging to H.

The question is how much the game changes if, as part of their strategizing, Alice and Bob prepare some quantum state on which they can later perform measurements. We say the graphs G and H are *quantum isomorphic* if there is a way for Alice and Bob to fool the referee with this additional resource.

We show that any two Hadamard graphs on the same number of vertices are quantum isomorphic. This follows from a more general recipe for showing quantum isomorphism involving exactly triply regular association schemes. I will briefly outline how the main result is built from three tools. A remarkable recent result of Mančinska and Roberson shows that graphs G and H are quantum isomorphic if and only if, for any planar graph F, the number of graph homomorphisms from F to G is equal to the number of graph homomorphisms from F to H. A generalization of partition functions used by Terwilliger and others (which I call "scaffolds") affords some basic reduction rules such as series-parallel reduction and can be applied to counting homomorphisms. The final tool is the classical theorem of Epifanov showing that any plane graph can be reduced to a single vertex and no edges by extended series-parallel reductions and Delta-Wye transformations.

The focus of this talk will be on the concepts; proofs will be omitted. We will use a simplified notion of quantum measurement and view this only through a linear-algebraic lens.

Graphs that support a uniform structure

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The notion of a uniform poset was introduced in 1990 by Terwilliger [2]. Vaguely speaking, a graded poset P is uniform, if the raising matrix and the lowering matrix of P satisfy certain linear dependencies. The notion of uniform poset could be easily adopted by bipartite graphs, as bipartite graphs can be viewed as Hasse diagrams for the graded posets. Uniform structures of Q-polynomial bipartite distance-regular graphs were studied in details in [1].

Assume now that Γ is a non-bipartite graph, and pick a vertex x of Γ. In this talk we first define what it means for Γ to *support a uniform structure with respect to* x. In case when Γ supports a uniform structure with respect to x we discuss algebraic properties of the corresponding Terwilliger algebra $T = T(x)$.

References:

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Distance-regular graphs with classical parameters that support a uniform structure: case $q \geq 2$

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With reference to the contributions of R. Maleki and Š. Miklavič, this talk will illustrate whenever a 1-thin distance-regular graph Γ with classical parameters (D, q, α, β), $D \geq 4$ and $q \geq 2$, supports a uniform structure (w.r.t. a fixed vertex of Γ). By [2] and [3], in order that the latter property is satisfied, such a graph Γ must admit exactly two (thin) irreducible T-modules with endpoint 1 (one with diameter $D-2$ and the other with diameter $D-1$), up to isomorphism. The analysis which arises from this consideration shows that for $\alpha \neq 0$ there remain only two feasible infinite families, whose respective classical parameters are

$$\Big(D, q, q, \frac{q^2(q^D-1)}{q-1}\Big), \qquad \Big(D, q, q+1, \frac{q^{D+1}(q+1)-q^2-1}{q-1}\Big).$$

Concerning the case $\alpha = 0$, examples are dual polar graphs, which are known to support a uniform structure [4, Proposition 26.4(i)]. Additionally assuming that Γ is a regular near polygon, it follows from [1, Theorem 9.4.4] that Γ is in fact a dual polar graph.

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The Generalized Terwilliger Algebra of the Hypercube

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For any distance-regular graph, there is a natural surjective homomorphism from its Generalized Terwilliger algebra to its Terwilliger algebra. It is known that in general, the map is not an isomorphism. However in some cases, this map is an isomorphism. In this talk, we will show that the map is an isomorphism if the graph is a hypercube.

On commutative association schemes and associated (un)directed family of graphs

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In this talk, we give an answer to the following problem:

Problem. Can the Bose-Mesner algebra $\mathcal{M}$ of every commutative d-class association scheme $\mathfrak{X}$ (which is not necessarily symmetric) be generated by a 01-matrix A? With other words, for a given $\mathfrak{X}$ can we find 01-matrix A such that $\mathcal{M} = (\langle A\rangle, +, \cdot)$? Moreover, since such a matrix A is the adjacency matrix of some (un)directed graph Γ, can we describe the combinatorial structure of Γ? Vice-versa question is also of importance, i.e., what combinatorial structure does (un)directed graph need to have so that its adjacency matrix will generate the Bose-Mesner algebra of a commutative d-class association scheme $\mathfrak{X}$?

Intersection density of vertex-transitive graphs

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A set of permutations $\mathcal{F}$ of a finite transitive group $G \leq \mathrm{Sym}(\Omega)$ is *intersecting* if any two permutations in $\mathcal{F}$ agree on an element of Ω. The *intersection density* of the transitive group G is the rational number

$$\rho(G) = \max\left\{ \frac{|\mathcal{F}|}{|G|/|\Omega|} \,:\, \mathcal{F} \subset G \text{ is intersecting} \right\}.$$

If $X = (V, E)$ is a vertex-transitive graph, then the *(intersection) density array* of X is the sequence of rational numbers $\rho(X) = [\rho_1, \rho_2, \ldots, \rho_t]$, where $\rho_1 < \rho_2 < \ldots < \rho_t$ and for any transitive subgroup $G \leq \mathrm{Aut}(X)$, there exists $i \in \{1, 2, \ldots, t\}$ such that $\rho(G) = \rho_i$.

In this talk, I will present some recent results on the intersection density array of Kneser graphs.

A uniform approach to the Damiani, Beck, and alternating PBW bases for the positive part of $U_q(\widehat{\mathfrak{sl}}_2)$

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This paper is about the positive part U_q^+ of the q-deformed enveloping algebra $U_q(\widehat{\mathfrak{sl}}_2)$. The algebra U_q^+ comes up in the theory of the association scheme on the bilinear forms. The literature contains at least three PBW bases for U_q^+, called the Damiani, the Beck, and the alternating PBW bases. These PBW bases are related via exponential formulas. In this paper, we introduce an exponential generating function whose argument is a power series involving the Beck PBW basis and an integer parameter m. The cases $m = 2$ and $m = -1$ yield the known exponential formulas for the Damiani and alternating PBW bases, respectively. The case $m = 1$ appears in the author's previous paper. In the present paper, we give a comprehensive study of the generating function for an arbitrary integer m. We have two main results. The first main result gives a factorization of the generating function. In the second main result, we express the coefficients of the generating function in closed form.

Designs in finite general linear groups

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It is known that the notion of a transitive subgroup of a permutation group G extends naturally to subsets of G. This talk is about subsets of the general linear group $\mathrm{GL}(n, q)$ acting transitively on flag-like structures, which are common generalisations of t-dimensional subspaces of $\mathbb{F}_q^n$ and bases of t-dimensional subspaces of $\mathbb{F}_q^n$. I shall discuss structural characterisations of transitive subsets of $\mathrm{GL}(n, q)$ using the character theory of $\mathrm{GL}(n, q)$ and interprete such subsets as designs in the conjugacy class association scheme of $\mathrm{GL}(n, q)$. While transitive subgroups of $\mathrm{GL}(n, q)$ are quite rare, it will be shown that, for all fixed t, there exist nontrivial subsets of $\mathrm{GL}(n, q)$ that are transitive on linearly independent t-tuples of $\mathbb{F}_q^n$, which also shows the existence of nontrivial subsets of $\mathrm{GL}(n, q)$ that are transitive on more general flag-like structures. These results can be interpreted as q-analogs of corresponding results for the symmetric group.

This is joint work with Alena Ernst.

Using a Grassmann graph to recover the underlying projective geometry

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In this talk we consider a type of distance-regular graph called a Grassmann graph. A Grassmann graph is constructed from a projective geometry. The possibility of recovering the whole projective geometry from a given Grassmann graph is still an uncertainty. Our goal is to present a number of results that describe the projective geometry in terms of the Grassmann graph.

Ruzsa triangle inequality for coherent configurations and diameters of vertex-transitive graphs

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We give a generalization of Ruzsa triangle inequality from additive combinatorics in the setting of homogeneous coherent configurations. As an application, we show that the oriented diameter of a vertex-transitive graph on n vertices can be bounded polynomially in terms of $\log n$ and its undirected diameter. This answers in the positive a question of L. Babai.

Quantum isomorphisms

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You will learn about quantum isomorphisms and new results in this area.

Hoffman graphs and Integral lattices

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It is well known that the smallest eigenvalue of a line graph is greater than or equal to -2.This naturally leads to the problem of knowing the hierarchical structure of graphs by the smallest eigenvalue, but the (well-known) method of constructing line graphs does not allow us to construct graphs with the smallest eigenvalue less than -2. Therefore, R. Woo and A. Neumaier (1995) introduced a construction method for graphs with the smallest eigenvalue less than 2 by generalizing well-known line graphs highly. This is where the relationship with the root system (2-lattice) arises, along with the irreducibility of Hoffman graphs. Using the idea of Hoffman graphs, a 3-lattice can be obtained from a Hoffman graph with the smallest eigenvalue greater than or equal to -3. However, nothing is known about 3-lattices that cannot be represented by Hoffman graphs. In studying this problem, we were able to find a generalization of Hoffman graphs. This will lead to contributions to the decomposition of directed graphs and non-simple graphs with vertex and edge weights, as well as to their (weighted) Hermitian and Seidel adjacency matrices. In this talk, we will talk on generalized Hoffman graphs and integral lattices.

The $\mathbb{Z}_3$-symmetric down-up algebra

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Recall that a dual polar graph Γ is distance-regular and Q-polynomial. With respect to a vertex x of Γ, the raising and lowering matrices satisfy a pair of relations called the down-up relations. The down-up algebra of Benkart and Roby is defined by two generators and the down-up relations. In this talk, we introduce a generalization of the down-up algebra that is $\mathbb{Z}_3$-symmetric. The $\mathbb{Z}_3$-symmetric down-up algebra is defined by three generators, any two of which satisfy the down-up relations. We discuss its representation theory and give some examples.

Bivariate P-polynomial association schemes

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Coauthors: Pierre-Antoine Bernard, Nicolas Crampé, Loïc Poulain d'Andecy, Meri Zaimi

Bivariate P-polynomial association schemes of type (α, β) are defined as a generalization of the P-polynomial association schemes. This generalization is shown to be equivalent to a set of conditions on the intersection parameters. A number of known higher rank association schemes are seen to belong to this broad class. Bivariate Q-polynomial association schemes are similarly defined.

Terwilliger algebras of generalized wreath products of association schemes

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The generalized wreath product of association schemes was introduced by R. A. Bailey in European Journal of Combinatorics 27 (2006) 428–435. It is known as a generalization of both wreath and direct products of association schemes. In this talk, we discuss the Terwilliger algebra of the generalized wreath product of commutative association schemes. I will describe its structure and its central primitive idempotents in terms of the parameters of each factors and their central primitive idempotents.

Packings and Steiner systems in polar spaces

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Coauthor: Kai-Uwe Schmidt

A finite classical polar space of rank n consists of the totally isotropic subspaces of a finite vector space equipped with a nondegenerate form such that n is the maximal dimension of such a subspace. A t-Steiner system in a finite classical polar space of rank n is a collection Y of totally isotropic n-spaces such that each totally isotropic t-space is contained in exactly one member of Y. Nontrivial examples are known only for $t = 1$ and $t = n - 1$. We give an almost complete classification of such t-Steiner systems, showing that such objects can only exist in some corner cases. This classification result arises from a more general result on packings in polar spaces, which we obtain by studying the association scheme arising from polar spaces and applying the powerful linear programming method from Delsarte.

A connection behind the Terwilliger algebras of the hypercube and its halved graph

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In 2002, Junie T. Go gave a connection between the Terwilliger algebra of the hypercube and the universal enveloping algebra $U(\mathfrak{sl}_2)$ of $\mathfrak{sl}_2$. In this talk, I will present my recent work on a connection between the universal Hahn algebra and $U(\mathfrak{sl}_2)$. Subsequently, I will explain the connection of our results with the Terwilliger algebras of the hypercube and its halved graph. This is based on joint work with Prof. Hau-Wen Huang.

On the size of maximal binary codes with 2, 3, and 4 distances

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We address the maximum size of binary codes and binary constant weight codes with few distances. Previous works established a number of bounds for these quantities as well as the exact values for a range of small code lengths. As our main results, we determine the exact size of maximal binary codes with two distances for all lengths $n \geq 6$ as well as the exact size of maximal binary constant weight codes with 2,3, and 4 distances for several values of the weight and for all but small lengths.

Bivariate P- and Q-polynomial structures for the non-binary Johnson scheme

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The non-binary Johnson scheme is a generalization of the Johnson scheme that involves bivariate polynomials in the expression of its eigenvalues and dual eigenvalues. As is known, the Johnson scheme belongs to the important class of (univariate) P- and Q-polynomial association schemes. Recently, the P- and Q-polynomial properties have been generalized to the bivariate case and multivariate case. In this talk, I will explain that the non-binary Johnson schemes possess both a bivariate P-polynomial structure and a bivariate Q-polynomial structure. I will also discuss the bispectrality of the bivariate polynomials, and some related algebras.

Discrete Quantum Walks in Association Schemes

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Discrete quantum walks are motivated by search problems. Unlike classical random walks, a discrete quantum walk usually takes place on the arcs of a graph, and are built upon shift and coin operators. This makes it difficult to relate properties of the walk to properties of the underlying graph.

In this talk, I will show that, despite the aforementioned difficulty, when the graph lies in some association schemes, we can say a lot about the walk using the adjacency spectrum of the graph. Part of this based on my joint book, Discrete Quantum Walks on Graphs and Digraphs, with Chris Godsil. No knowledge of quantum physics is required.

Oriented or signed Cayley graphs with nice eigenvalues

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Let G be a finite abelian group. Bridges and Mena gave a characterization when a Cayley graph on G has only integer eigenvalues. Here we consider oriented or signed Cayley graphs on G, whose adjacency matrices are skew symmetric or symmetric (0,1,-1) matrices, respectively. We give a characterization of when all the eigenvalues of such a graph are integer multiples of $\sqrt{\Delta}$ for some square-free integer Δ. This also characterizes oriented or signed Cayley graphs on which continuous quantum walks are periodic. In particular, in an oriented graph, periodicity is a necessary condition for the occurrence of uniform mixing or perfect state transfer.

Quantum Algebra via q-difference Operators and Its Representations

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Recently, Ismail, Zhang and Zhou introduced two q-difference operators $\mathcal{B}_q$ and $\mathcal{C}_q$. Considering those two operators and the Askey-Wilson operator $\mathcal{D}_q$, we get a new quantum algebra, denoted U_q. In this paper, we display a basis for the vector space U_q. We show that the center of U_q is spanned by the identity 1, provided that q is not a root of 1. We give a connection between U_q and $U_{q^{1/2}}(sl_2)$. We study the finite-dimensional and infinite-dimensional irreducible U_q-modules. For both types of modules, a full classification appears to be difficult and is not contained in the paper. However, we have the following results. We use bidiagonal pairs and lowering-raising triples to construct some finite-dimensional irreducible U_q-modules. We give an example of an infinite-dimensional U_q-module. For this module, the underlying vector space is a Hilbert space with respect to the weight function for the continuous q-Hermite polynomials.

Euclidean t-designs from the spherical embedding of coherent configurations

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The criterion for the spherical embedding of a d-class Q-polynomial association scheme to be a spherical t-design was given by Munemasa (in 2004) for $t \leq 5$ in terms of the Krein parameters. Suda generalized the result for any fixed $t \leq 2d$ and obtained an upper bound for the strength t of a P- and Q-polynomial association scheme as a spherical t-design.

The concept of Q-polynomial coherent configurations was introduced by Suda in 2021, which is a generalization of Q-polynomial association schemes. In this talk, we will discuss the necessary and sufficient conditions (using the Krein numbers) that the embedding of a Q-polynomial coherent configuration becomes a Euclidean t-design for some t. We will also give some examples of Euclidean 2- or 3-designs from the spherical embedding.

Twin Buildings and Hypergroups

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It is well known that each building is mathematically equivalent to a regular action of a Coxeter hypergroup. In my talk, I will show to which extend a similar statement holds for twin buildings.

Minisymposium

Chemical Graph Theory

Organized by Nino Bašić, *University of Primorska*

INVITED TALK

Nut graphs in chemistry

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Nut graphs have the property that they have a single non-trivial eigenvector, and this unique vector is full, i.e. it has no zero entries. Remarkably, this simple definition has significant consequences for the chemistry of π-conjugated systems. It gives a model for radicals with fully distributed spin density, and a model for the molecules with exactly one non-bonding π molecular orbital that are *omniconductors* in the simplest model of ballistic conduction. Signed nut graphs provide models for molecular structures with Möbius twists in their π systems. In this talk chemical applications, orders, degree sequences and constructions of chemical nut graphs will be discussed, and the chemical nut graphs will be placed in a more general context. This talk includes joint work with Nino Bašić, Martha Borg, Tomaž Pisanski, Barry Pickup, and Irene Sciriha.

INVITED TALK

Comparative analysis of three eigenvalue-based topological indices

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Graph energy and the Estrada index are well-established topological descriptors that have been frequently used in molecular modeling and QSPR/QSAR investigations. These indices were among the few that are based on the eigenvalues of the characteristic polynomial of a graph. Recently, another descriptor that belongs to this class of indices was put forward. It was named *resolvent energy*. There are a couple of papers dealing with this descriptor. Using Randić's list of qualities that a molecular descriptor should possess as a starting point, the results of the comparative analysis of these three invariants are going to be presented.

Contructing all fullerenes that have 3-fold rotational symmetry

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A fullerene is a spherical embedding of a cubic graph in which all faces are hexagonal or pentagonal. Working with dual graphs of fullerenes, we construct all fullerenes that have 3-fold rotational symmetry. These can be classified into two families: For one family we provide a multiparameter net construction where the nets are described in terms of how they are positioned on the regular tiling of the plane with equilateral triangles. For the other family the net construction is entirely unnatural, so we reduce the problem to classifying all graph embeddings in the disk having particular restrictions on vertex degree. We then describe how all such disk embeddings can be constructed using combinations of five moves from the unique minimal such embedding.

Nut graphs with a small number of vertex and edge orbits

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A nut graph has a single non-trivial kernel eigenvector and that vector contains no zero entries. If the isolated vertex is excluded as trivial, nut graphs have seven or more vertices; they are all connected, non-bipartite, and have no leaves. A nut graph may be vertex transitive; there are known examples of circulant nut graphs, Cayley nut graphs, and also non-Cayley vertex-transitive nut graphs. We will show that no nut graph can be edge transitive. Furthermore, a nut graph always has strictly more edge orbits than vertex orbits. We also construct several families of nut graphs with a small number of vertex orbits and edge orbits as regular coverings over certain voltage graphs (using non-cyclic groups).

This is joint work with Patrick W. Fowler and Tomaž Pisanski.

A Vertex-Deleted Subgraph Reveals: An Inverse Problem

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Suppose we are given a graph G that is a one-vertex deleted subgraph of an unknown graph H. We study the question of which spectral properties of the adjacency matrix of H may be determined from G.

In particular, we find that if H has a prescribed eigenvalue μ that is not an eigenvalue of G, then there is a bijection between the set of non-isomorphic graphs H that have a G as a vertex-deleted subgraph, and the set of possible linearly independent one-dimensional μ-eigenspaces. We show how, for each of these eigenspaces, the corresponding unique graph H may be constructed.

Furthermore, if G has an eigenvalue μ of multiplicity 1, then the μ-eigenspace of G may be related to an eigenspace of H, depending on whether μ is an eigenvalue of H or not.

2-connected outerplane bipartite graphs with isomorphic resonance graphs

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Coauthors: Zhongyuan Che, Niko Tratnik, Petra Žigert Pleteršek

In the talk, we present novel results related to isomorphic resonance graphs of 2-connected outerplane bipartite graphs. We first show that two 2-connected outerplane bipartite graphs have isomorphic resonance graphs if and only if they can be properly two colored so that their resonance digraphs are isomorphic. Moreover, we prove that 2-connected outerplane bipartite graphs G and H have isomorphic resonance graphs if and only if there exists an isomorphism α between their inner duals T and T' such that for any 3-path xyz of T, the triple (x, y, z) is regular if and only if $(\alpha(x), \alpha(y), \alpha(z))$ is regular.

Peripheral Convex Expansions of Resonance graphs

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It is well known that the resonance graph of a plane elementary bipartite graph is a median graph. The most important structural characterization of a median graph is the Mulder's convex expansion theorem. Fibonacci cubes used in network designs are median graphs that can be obtained from an edge by a sequence of peripheral convex expansions. Plane bipartite graphs whose resonance graphs are Fibonacci cubes were studied by Klavžar et al. first, and characterized by Zhang et al. completely. Peripheral convex expansions of resonances graphs have been studied for catacondensed even ring systems by Klavžar et al., and for catacondensed hexagonal graphs by Vesel. We later showed that peripheral convex expansions hold for resonance graphs of 2-connected outerplane bipartite graphs. In this talk, we further characterize all plane bipartite graphs whose resonance graphs can be constructed from an edge by a sequence of peripheral convex expansions.

An algorithmic approach to extending a theorem on extremal trees by Wang

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Let $\mathcal{T}_D$ be the set containing all of the trees corresponding to a given degree sequence D and let $R_\mathcal{F}$ be the graph invariant defined via

$$R_\mathcal{F} = \sum_{u \sim v} \mathcal{F}(\deg(u), \deg(v)),$$

where the summing is performed across all the unordered pairs of adjacent vertices u and v, with $\mathcal{F} \colon \mathbb{N} \times \mathbb{N} \to \mathbb{R}$ being a symmetric function such that

$$\mathcal{F}(x, a) + \mathcal{F}(y, b) > \mathcal{F}(y, a) + \mathcal{F}(x, b) \quad \text{for any } x > y \text{ and } a > b.$$

In an earlier paper, Wang [*Cent. Eur. J. Math.* **12** (2014) 1656–1663] demonstrated that the greedy tree must always attain the maximum $R_\mathcal{F}$ value on $\mathcal{T}_D$, while an alternating greedy tree necessarily attains the minimum $R_\mathcal{F}$ value on $\mathcal{T}_D$. Here, we implement an algorithmic approach in order to find the full solution set to both the $R_\mathcal{F}$ maximization and $R_\mathcal{F}$ minimization problem on $\mathcal{T}_D$. In other words, we determine all the trees from $\mathcal{T}_D$ that attain the maximum $R_\mathcal{F}$ value on this set, together with all the trees from $\mathcal{T}_D$ that minimize the said graph invariant, thereby improving the aforementioned earlier result obtained by Wang.

On the Wiener Index of Orientations of Graphs

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The Wiener index $W(D)$ of a strong digraph D with vertex set V is defined by

$$W(D) = \sum_{(a,b)\in V\times V} d_D(a,b),$$

where $d_D(a,b)$ denotes the usual shortest path distance from a to b in D. This definition has been extended to the case that D is not strong by defining $d_D(a,b)$ as 0 if there is no path from a to b in D.

In our talk we consider the Wiener index of (not necessarily strong) orientations of graphs. Knor, Škrekowski and Tepeh conjectured that every tree has an orientation of maximum Wiener index in which there is one vertex v with the property that for every vertex w there is either a path from w to v or a path from v to w. We disprove the conjecture. We also settle a question by the same authors on the computational complexity of finding an orientation of maximum Wiener index by showing that the corresponding decision problem is NP-complete. The problem of finding an orientation of minimum Wiener index of a given graph is discussed briefly.

Rule Inference and Maximal Common Subgraph in Chemical Reaction Networks

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There are several abstract formalisms for graph transformations in the literature that have been successfully applied to expand metabolic networks and chemical reaction networks. Among them, we are interested in the *Double Push-Out* approach. A *DPO transformation rule* $p = (L \overset{l}{\leftarrow} K \overset{r}{\rightarrow} R)$ consists of three graphs L, R and K known as the left, right and context graph, respectively, and two graph morphisms l and r that determine how the context is embedded in the left and the right graph. The rule p can be applied to a graph G if there is a *matching morphism* $m : L \to G$ describing how L is contained in G. This operation is called *derivation* and is denoted by $G \overset{p,m}{\Longrightarrow} H$.

Assume that a chemical reaction network $L := \{R_1, R_2, \ldots, R_n\}$ is given. Also, an atom map $f_i : G_i \to H_i$, for every reaction $R_i : G_i \longrightarrow H_i$ is given. The aim is to describe the network with a minimum size set of subrules. In other words, the goal is to find a set of rules which has the minimum size and if they are applied to the educts, then the original network is produced. In this talk, we will see how the maximal common subgraph problem is related to that. Moreover, we will briefly discuss how a maximal common subgraph algorithm can be modified for this special application to handle the bottleneck of the original problem.

What's the price of being nice (in fullerene graphs)

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Let G be a graph with a perfect matching. A subgraph H of G is nice if $G - V(H)$ still has a perfect matching. In a chemical context, nice subgraphs of molecular graphs serve as mathematical models of addition patterns in the corresponding molecules such that the rest of the molecule still has a resonant structure. In this contribution we consider classical and generalized fullerene graphs and look for nice subgraphs with prescribed components such as, e.g., stars and odd cycles. We also report some computational results for small fullerenes and list some open problems.

Joint work with Meysam Taheri-Dehkordi and Gholam Hossein Fath-Tabar of Kashan, Iran.

Connected graphs with maximal Graovac-Ghorbani index

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Based on a computer search, Furtula [1] characterized the connected graphs with maximal ABC_{GG} index. In this talk, we present a mathematical proof of the established hypothesizes.

References:

[1] B. Furtula, Atom-Bond connectivity index versus Graovac-Ghorbani analog, *MATCH Commun. Math. Comput. Chem.* **75** (2016) 233-242.

Results on the Clar numbers of fullerenes

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A *fullerene* is a 3-regular plane graph with only hexagonal and pentagonal faces, and models a pure carbon molecule. A *Kekulé structure* of a fullerene is a perfect matching of its edges, and represents a double bond structure for the molecule. A *benzene ring* is a hexagonal face with 3 of its bounding edges in the Kekulé structure. The *Clar number* of a fullerene is the maximum number of independent benzene rings over all possible Kekulé structures, and is related to the aromaticity and stability of the molecule. In this talk, we describe several recent results about the Clar numbers of fullerenes, especially for nanotubes and leapfrog fullerenes. Several open problems will be discussed.

Advancing Drug Design through ProBiS: Leveraging Protein Product Graphs

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We have successfully developed an extensive suite of protein binding site tools called ProBiS. This comprehensive suite encompasses web servers, databases, and algorithms that excel in predicting protein binding sites and ligands. At the heart of ProBiS lies a novel algorithm based on Protein Product Graphs, which harnesses graph theory principles to achieve rapid and accurate identification of the largest fully connected subgraph within a product protein graph.

Building upon this innovative framework, we have made remarkable strides in tackling various pharmaceutical challenges using our ProBiS tools, which currently stand as the most precise and efficient solutions available. To expand the impact of our approach, we have extended the protein interaction network beyond the Protein Data Bank (PDB) structures. By integrating structures predicted with the AlphaFold method, we have augmented the network to encompass over 20,000 human proteins, as well as other biologically significant organisms. Consequently, this expanded protein interaction network is projected to comprise millions of predicted interactions, comprising both experimentally confirmed and undiscovered interactions.

With a long-term vision in mind, we aim to transform our approach into a personalized medicine paradigm, capable of addressing the forefront challenges in modern research. By leveraging the power of protein product graphs and the extensive protein interaction network, we anticipate our work will contribute significantly to advancing the field of drug design and pave the way for cutting-edge discoveries in personalized medicine.

The Cut Method on Hypergraphs for the Wiener Index

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In this talk, we explore the application of the cut method in chemical graph theory. While it may be more suitable to represent a chemical molecule using a hypergraph in certain scenarios, we propose an extension of the standard cut method to handle hypergraphs. Our focus lies on the introduction of partial-cube hypergraphs, which serve as analogues to partial cubes. Additionally, we demonstrate the adaptation of this method to hypergraphs arising in chemistry that may not necessarily be k-uniform or linear.

Uniform Core Graphs

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A graph G is singular of *nullity* η if the nullspace of its 0–1–adjacency matrix $\mathbf{A}$ has dimension $\eta \geq 1$. Such a graph contains η *cores* determined by the non–zero entries of the vectors in a basis for the nullspace of $\mathbf{A}$. In a *core graph*, the non–zero entries cover all the vertices. A *uniform core–graph* (UCG) is a core graph whose nullity reduces by two on deleting any two vertices. They are the only omni–insulating molecular devices among core graphs. We explore the properties of UCGs and generate some families.

On Distance and Degree-Based Topological Characterization of Nanodiamond Structure

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Nanodiamonds are new nanoscale carbon building blocks with a wide range of intriguing mechanical, chemical, optical, and biological properties, making them essential functional moiety

carriers for drug delivery. These crystals are considered the most captivating due to their chemical barrier and unique properties, which make them suitable for a wide range of biomedical applications. To fully exploit nanodiamond's drug-delivery ability, attention must be paid to its purity, surface chemistry, and other properties that can affect drug adsorption on nanodiamonds and drug release in the biological environment, either directly or indirectly. The structure-property relationship is a quantitative relationship between chemical structural features known as molecular descriptors and pharmacological activity, used as response endpoints. The topological index is a molecular descriptor widely used in the study of the structure-property relationship of pharmaceuticals to calculate their molecular characteristics numerically. In this article, we calculate the various distance, degree and bond additive topological indices of nanodiamond structure.

The Generalized Zhang-Zhang Polynomial of Benzenoid Systems

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The generalized Zhang-Zhang polynomial was introduced in 2018 aiming to increase the sensitivity of the well-known Zhang-Zhang polynomial by taking into account also 10-cycles of a given molecular graph. In this talk, we will present some recursive formulas for the calculation of the generalized Zhang-Zhang polynomial of benzenoid systems. More precisely, such results enable us to compute the generalized Zhang-Zhang polynomial of a benzenoid system by using its subgraphs. Then, we will show an algorithm for calculating the generalized Zhang-Zhang polynomial of benzenoid chains. Finally, chemical applicability of the generalized Zhang-Zhang polynomial will be discussed.

Partition of topological indices of benzenoid hydrocarbons into cycles contributions

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We present a simple method for partitioning the bond-additive and atoms-pair-additive distance-based topological indices of plane graphs into the sum of contributions of inner faces. The proposed method is applied to decompose several topological indices (Wiener, hyper-Wiener, Tratch-Stankevich-Zefirov, Balaban, and Szeged indices) into the ring contributions for a series of benzenoid systems. It was found that the employed ring partitioning scheme is providing an accurate assessment of the dominant cyclic conjugation modes in the studied benzenoid hydrocarbons. Thus, the proposed method can be used as the alternative for the quantum-chemistry-based aromaticity indices which are significantly more computationally demanding.

Minisymposium

Combinatorial Designs And Their Applications

Organized by Anita Pasotti, *University of Brescia*
Coorganized by Tommaso Traetta, *University of Brescia*

INVITED TALK

Combinatorial Designs and Colouring

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We will begin this talk with a brief introduction to combinatorial designs and graph decompositions. In general, a graph decomposition or a G-design of order v consists of a collection $\mathcal{B}$ of subgraphs of the complete graph K_v such that each subgraph in $\mathcal{B}$ is isomorphic to G, and the subgraphs of $\mathcal{B}$ partition the edge set of K_v. Among the earliest designs to be studied were what are now known as Steiner triple systems, namely C_3-decompositions of complete graphs.

When referring to a G-design, each copy of G in $\mathcal{B}$ is known as a block, and the vertices of the complete graph K_v that is being decomposed are generally called points.

Combinatorial designs have a rich history with many interesting properties and applications. In this presentation we will focus on aspects of colourings of designs whereby each point is assigned a colour in such a way that certain properties are ensured, such as requiring that each block has at least two colours among its points. We will survey some of the history of such colourings, along with presenting recent results and open problems.

Grid-Based Graphs, Linear Realizations, and the Buratti-Horak-Rosa Conjecture

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Label the vertices of the complete graph K_v with the integers $\{0, 1, \ldots, v-1\}$ and define the *length* ℓ of the edge between distinct vertices labeled x and y by $\ell(x, y) = \min(|y-x|, v-|y-x|)$. A *realization* of a multiset L of size $v-1$ is a Hamiltonian path through K_v whose edge labels are L. The *Buratti-Horak-Rosa (BHR) Conjecture* is that there is a realization for a multiset L if and only if for any divisor d of v the number of multiples of d in L is at most $v-d$.

We introduce "grid-based graphs" as a powerful tool for constructing particular types of realizations, called "linear realizations", especially when the multiset in question has a support of size 3. This lets us prove many new instances of the BHR Conjecture, including those for all multisets of the following forms for sufficiently large v with $\gcd(v, y) = 1$ for all $y \in L$:

- $\{1^a, 2^b, x^c\}$, except possibly when $a \in \{1, 2\}$ and x is odd,
- $\{1^a, x^b, (x+1)^b\}$.

This shows that infinitely many multisets with support of size larger than 2 satisfy the BHR Conjecture for infinitely many values v for the first time.

As well as the above specific results, we discuss how pursuing these methods may prove the conjecture for increasingly general families of parameters.

Symmetric Layer-Rainbow Colorations of Cubes

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Can we color the n^3 cells of an $n \times n \times n$ cube L with n^2 colors in such a way that each layer parallel to each face contains each color exactly once and that the coloring is symmetric so that $L_{ij\ell} = L_{j\ell i} = L_{\ell ij}$ for distinct $i, j, \ell \in \{1, \ldots, n\}$, and $L_{iij} = L_{jji}, L_{iji} = L_{jij}, L_{ijj} = L_{jii}$ for $i, j \in \{1, \ldots, n\}$? Using transportation networks, we show that such a coloring is possible if and only if $n \equiv 0, 2 \pmod 3$ (with two exceptions; n can be 1, but n cannot be 3). Motivated by the designs of experiments, the study of these objects (without symmetry) was initiated by Kishen and Fisher in the 1940's. These objects are also closely related to orthogonal arrays whose existence has been extensively investigated, and they are natural three-dimensional analogues of symmetric latin squares.

Rank-one Heffter arrays and their descendants

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A Heffter array $H(m, n)$ in an abelian group G of order $2mn + 1$ is a $m \times n$ matrix whose elements form a complete system of representatives for the *patterned starter* of G and whose rows and columns are all zero-sum.

Cyclic Heffter arrays have been studied by many authors [4]. In particular, their existence was proved without any exception in [1]. I recently considered elementary abelian Heffter arrays, hence with entries in a finite field, focusing my attention on those having rank one in the sense of linear algebra [2]. In this talk I will first speak about their existence. Then I will show how a rank-one $H(n, n)$ may have many zero-sum transversals [3] leading sometimes to useful so-called *Heffter configurations*.

References:

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[2] M. Buratti, *Tight Heffter arrays from finite fields*, to appear in Fields Institute Communications.

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Equitably colourable cycle systems

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An ℓ-cycle decomposition of a graph G is said to be *equitably c-colourable* if there is a c-vertex colouring in which each colour occurs either $\lfloor \ell/c \rfloor$ or $\lceil \ell/c \rceil$ times in each cycle, that is, each colour is represented (as closely as possible) an equal number of times on each cycle. Equitably colourable cycle systems were introduced in work by Adams, Bryant, Lefevre and Waterhouse, who considered equitably 2- and 3-colourable ℓ-cycle systems of K_v and $K_v - I$ for $\ell \in \{4, 5, 6\}$. In this talk, we discuss some constructions of equitably 2-colourable ℓ-cycle decompositions of K_v and $K_v - I$ in the case that v and ℓ have the same parity.

Conditions for the Block Intersection Graph (BIG) of Packings and Coverings to be Hamiltonian

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A double change covering design is a sequence of b k-sets, called blocks, of a V-set in which exactly two elements differ between consecutive blocks and every pair of elements in V is in some block.

We determine sufficient conditions for the block intersection graph (BIG) of block size k packings and block size 3 coverings to be Hamiltonian. The BIG of a packing is Hamiltonian for k even if $4[|X||V\backslash X| - \partial(X)] \geq vk$ and for k odd if $4k[|X||V\backslash X| - \partial(X) \geq v(k^2-1)$. The BIG of a covering is Hamiltonian if $v \geq 3$. Because of our interest in DCCD, we are also interested in Hamiltonian cycles in 1-BIG of block size 3 coverings and we discuss our progress in this case.

Covering Arrays: Many Hands Make Light Work

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Except when the number of symbols is very small, the best probabilistic bounds on sizes of covering arrays (of index one) and the best current algorithms for their construction both employ a compact representation using vectors over the finite field. Covering perfect hash families, which provide this compact representation, have recently been generalized to index greater than one. Unfortunately, the naive probabilistic bound degrades as the index increases, because it treats each event as totally covered or not covered at all. By accounting for rows that provide partial coverage, we improve the probabilistic analysis. In the process we demonstrate that covering perfect hash families can yield competitive bounds on covering array sizes when the index is large. Most surprisingly, the same approach improves on the bounds when the index is one and the number of symbols is small.

Weak Sequenceability in Generic Groups

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A subset A of a group G is *sequenceable* if there is an ordering $(a_1, \ldots, a_k)$ of its elements such that the partial sums $(s_0, s_1, \ldots, s_k)$, given by $s_0 = 0$ and $s_i = \sum_{j=1}^{i} a_j$ for $1 \leq i \leq k$, are distinct, with the possible exception that we may have $s_k = s_0 = 0$. Several conjectures and questions concerning the sequenceability of subsets of groups arose in the Design Theory context: indeed this problem is related to Heffter Arrays and Graph Decompositions. Alspach and Liversidge combined and summarized many of them into the conjecture that if a subset of an abelian group does not contain 0 then it is sequenceable.

Note that, if the elements of a sequenceable set A do not sum to 0, then there exists a simple path P in the Cayley graph $Cay[G : \pm A]$ such that $\Delta(P) = \pm A$. In this talk, inspired by this interpretation, we propose a weakening of this conjecture. Here we want to find an ordering whose partial sums define a walk W of girth bigger than t (for a given $t < k$) and such that $\Delta(W) = \pm A$. This is possible given that the partial sums s_i and s_j are different whenever i and j are distinct and $|i - j| \leq t$. In this case, we say that the set A is t-*weakly sequenceable*. The main result we will present is that, if the size of a subset A of a (not necessarily abelian) group G is big enough and if A does not contain 0, then A is t-weakly sequenceable. This result has been obtained using a hybrid approach that combines both Ramsey theory and the probabilistic method.

References:
[1] B. Alspach and G. Liversidge, On strongly sequenceable abelian groups, *Art Discrete Appl. Math.* **3** (2020) 19.
[2] S. Costa, S. Della Fiore, Weak Sequenceability in Cyclic Groups, *J. Comb. Des.* **30** (2022) 735-751.

Deza digraphs

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Deza digraphs were introduced in 2003 by Zhang and Wang, as a directed graph version of Deza graphs, that also generalize the notion of directed strongly regular graphs. In this talk we give new constructions of Deza digraphs. Further, we present examples of twin and Siamese twin (directed) Deza graphs constructed using Hadamard Matrices. Finally, we present a variation of directed Deza graphs, called directed Deza graph of type II, and provide a construction from finite fields.

On Tight Cycle and Other Hypergraph Designs

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A common question in combinatorics pertains to the decompositions of graphs into edge-disjoint subgraphs. For graphs G and K, a G-decomposition of K is a partition of the edge set of K into subgraphs isomorphic to G. Such decompositions are generally known as graph designs. The problem is of most interest when both G and K are complete graphs. Other cases though have also attracted attention. Decompositions of complete graphs into cycles is a celebrated problem.

Corresponding questions for decompositions of t-uniform hypergraphs are of interest. In a t-uniform hypergraph, the edges are t-element subsets of a vertex set (thus graphs correspond to the case $t = 2$). The complete t-uniform hypergraph of order v, denoted $K_v^{(t)}$, has a set V of size v as its vertex set and the set of all t-element subsets of V as its edge set. For $t > 2$, the only general question on H-decompositions of $K_v^{(t)}$ that is settled completely is the case where H is a matching. Though, it is now known that the necessary conditions for H-decompositions of $K_v^{(t)}$ are asymptotically sufficient for all t-uniform hypergraphs H.

A seminal 2014 paper by Bryant, Herke, Maenhaut, and Wannasit [Decompositions of complete 3-uniform hypergraphs into small 3-uniform hypergraphs, *Australas. J. Combin.* **60** (2014), 227–254] gives a template for constructing 3-uniform hypergraph designs. Since 2019, participants in the *Illinois State University Math REU for Pre-service and In-service Teachers* have used the Bryant et al approach to settle H-decompositions of complete and λ-fold complete 3-uniform hypergraphs for several choices of H, including tight 6-cycles and tight 9-cycles and multiple other regular cycle-like hypergraphs. They have also investigated maximum packings of 3-uniform λ-fold complete hypergraphs with 2-regular cycle-like structures and with the lines of the Pasch configuration. Investigations have also included decompositions of complete 4-uniform hypergraphs into cycle-like structures, including two 2-regular four-edge ones.

We report on some of the results from the Illinois State REU and highlight how the approach by Bryant et al is used to obtain them. Parts of these types of problems are highly suitable for working with undergraduates and with high school teachers.

Tightness of the weight-distribution bound for polar and affine polar graphs

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The *weight-distribution bound* is a lower bound on the cardinality of support of an eigenfunction of a distance-regular graph with given intersection numbers. Krotov et al. introduce this bound to study a general class of combinatorial bitrades. In the study of the weight-distribution bound for Paley graphs of square order, Goryainov et al. reduce the classification of eigenfunctions with minimal support to the classification of maximal cliques of a certain size. This approach has been extended to the study eigenfunctions of generalised Paley graphs.

Motivated by the connections between cliques and eigenfunctions, we study the weight-distribution bound for classes of graphs related to polar spaces. First we introduce the weight-distribution bound for distance-regular graphs, and focus on the weight-distribution bound for strongly regular graphs. Then we introduce polar graphs, a class of strongly regular graphs defined as the collinearity graphs of embedded polar spaces. For a polar graph Γ with second largest eigenvalue ρ, we characterise the ρ-eigenfunctions with cardinality of support equal to the weight-distribution bound. Further, for an affine polar graph Γ with second largest eigenvalue ρ, we extend our understanding of polar spaces to characterise the ρ-eigenfunctions with cardinality of support equal to the weight-distribution bound.

Constructions and applications of Disjoint Partial Difference Families and External Partial Difference Families

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A Disjoint Partial Difference Family (respectively External Partial Difference Family) is a collection of disjoint subsets of a group G, such that when you take the pairwise differences between distinct elements contained within the same subset (respectively pairwise differences between elements of distinct subsets) you produce each element contained within one of the component subsets λ times and all other non-identity elements of G μ times.

Recent work into these objects has centered around looking at the applications of these objects in design theory and information security, as well as looking for new constructions of these objects. In this talk, I will give an overview of the some of the recent work in this area.

A new family of 3-designs of degree 3

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We shall present a new family of 3-designs with three intersection numbers. The new designs are similar to a known family obtained from linked systems of symmetric designs (LSSDs) with the maximum number of fibers [1,2]. The LSSD family is block schematic, i.e. the set of blocks equipped with relations defined by intersection sizes is a three-class association scheme. The number of points is a power of 2 with even exponent. The new family is not block schematic and the number of points is a power of 2 with odd exponent.

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[2] J.-M. Goethals, *Nonlinear codes defined by quadratic forms over $GF(2)$*, Information and Control **31** (1976), no. 1, 43–74.

Resolution of the directed Oberwolfach Problem with cycles of equal length

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A $\vec{C}_m$-factor of a digraph is a spanning subdigraph comprised of disjoint directed cycles of length m and a $\vec{C}_m$-factorization is a decomposition into $\vec{C}_m$-factors. It has been conjectured that $K^*_{\alpha m}$ admits a $\vec{C}_m$-factorization if and only if $(\alpha, m) \notin \{(1,4),(1,6),(2,3)\}$. This problem is known as the directed Oberwolfach Problem with cycles of equal length. In this talk, we present a solution to the last outstanding case; that is, we show that K^*_{2m} admits a $\vec{C}_m$-factorization for all odd $m \geqslant 11$.

Circulant almost orthogonal arrays and related combinatorial designs

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This talk focuses on circulant almost orthogonal arrays (CAOAs), a type of circulant arrays used for designs of fMRI experiments that generalize circulant orthogonal arrays. We will concentrate on binary CAOAs with strength 2, from both aspects of combinatorics and statistical designs of experiments. Specifically, we will introduce new characterizations and constructions using combinatorial designs like difference sets and almost difference sets. In addition, we may discuss the challenges of constructing CAOAs, as well as some open problems.

Cycles and paths: largely blocked designs and perfect blocking sets

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Let Σ be a G-design. A *transversal* T of Σ is a subset of its vertex set intersecting every block of Σ. A *blocking set* T of Σ is a transversal such that also its complement is a transversal of Σ. The existence of possible blocking sets has been studied for t-designs, projective planes, symmetric designs, block designs, balanced and almost balanced path designs, and G-designs when G has fewer than 5 edges.
Gionfriddo-Milazzo gave the notion of *largely blocked* C_k-designs for $k \geq 4$. The idea is that the set of all possible $p \in \mathbb{N}$ for which there exist in Σ blocking sets of cardinality p has the maximum possible cardinality. Moreover, a blocking set T of Σ is called *perfect* if in any block the number of edges between elements of T and elements in the complement is equal to a constant. *Largely blocked* C_4, P_3-designs and *perfect* blocking sets for C_4 and P_5-designs will be illustrated together with old and new results.

On Weak Heffter Arrays

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Weak Heffter arrays were defined by Archdeacon in [1] as a variant of the concept of Heffter arrays (see [3] for a detailed survey on the topic), in order to obtain regular 2-colorable embeddings of complete graphs over non-orientable surfaces. In this talk we present results obtained in [2], that is the first paper on this topic. In particular we give necessary conditions, and existence and non-existence results for this class of arrays.

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Additive Graph Decompositions

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A $t-(v,k,\lambda)$ design is additive if, up to isomorphism, the point set is a subset of an abelian group G and every block is zero-sum. This definition was introduced in [2] and was the starting point of an interesting new theory, see also for instance [1, 3, 4].

One might generalize this concept in a natural way to graph decompositions as follows: an *additive graph decomposition* is a decomposition of a graph K into subgraphs $\Gamma_1, \Gamma_2, \ldots, \Gamma_t$ such that the vertex set $V(K)$ is a subset of an abelian group G and the sets $V(\Gamma_1), V(\Gamma_2), \ldots V(\Gamma_t)$ are zero-sum in G.

I will report on some work in progress on constructing additive graph decomposition, in particular in the cases of cycle and path decomposition of the complete graph.

References:

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Flag-transitive 2 -(v,k,λ) designs with $\lambda \geq \gcd(r,\lambda)^2$

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The classification of the pairs ($\mathcal{D}$,G), where $\mathcal{D}$ is a 2-(v,k,λ) design admitting G as a flag-transitive automorphism group, is a widely studied problem in Design Theory. If $\lambda = 1$, then G acts point-primitively on $\mathcal{D}$ by an old result of Higman and McLaughlin (1960), and the classification of ($\mathcal{D}$,G) was achieved by Buekenhout, Delandtsheer, Doyen, Kleidman, Liebeck and Saxl (1990) except when v is a power of a prime and $G \leq A\Gamma L_1(v)$. Although examples of ($\mathcal{D}$,G) with G acting flag-transitively and point-imprimitively on $\mathcal{D}$ do exist for $\lambda > 1$ as shown mainly by Praeger (2007), the action of G on the point-set of $\mathcal{D}$ is primitive when the parameters of this one fulfill suitable constraints such as $\lambda \geq \gcd(r,\lambda)^2 - \gcd(r,\lambda)$ or $\gcd(r,\lambda) = 1$ by a result of Dembowski (1968). The latter case has been completely solved by Alavi, Biliotti, Daneshkakh, Montinaro, Zhou et al. (2022) except when v is a power of a prime and $G \leq A\Gamma L_1(v)$, whereas the former is essentially open. However, if $\lambda \geq \gcd(r,\lambda)^2$ then G is either almost simple or of affine type by a result of Li, Zhan and Zhou (2023) relying on the O'Nan-Scott theorem.

In this talk we analyze the pairs ($\mathcal{D}$,G) with $\lambda \geq \gcd(r,\lambda)^2$ and G of affine type, thus continuing the work started by Zhou et al. In particular, we show that we are towards a classification of ($\mathcal{D}$,G), again except when v is a power of a prime and $G \leq A\Gamma L_1(v)$, by combining some tools arising from Finite Geometry, such as the Segre varieties, together with an adaptation to our context of some group-theoretic arguments developed by Liebeck (1998) for flag-transitive linear spaces. Finally, when $G \leq A\Gamma L_1(v)$ we provide some new remarkable examples which are obtained by gluing suitable translation planes admitting a solvable flag-transitive collineation group.

New results on additive designs

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The interesting topic of *additive designs* was recently introduced by Caggegi, Falcone and Pavone [2]. A 2-(v, k, λ) design $(V, \mathcal{B})$ is *additive* under an abelian group G if there exists an injection $f : V \longrightarrow G$ mapping every block $B \in \mathcal{B}$ in a zero-sum subset of G. We say that it is *strictly additive* when f is a bijection.

In this talk I would like first to recall what is known on the strictly additive Steiner 2-designs [1]. I will then present new results obtained in a joint work still in progress with Marco Buratti about the additivity of PG$_d(n, q)$ (the design of points and d-dimensional subspaces of the projective geometry PG(n, q)) and about the additivity of cyclic symmetric designs.

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How to design a ghost

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A ghost is a (multi)subset of the projective plane PG$(2, q)$ with the property that its associated power sum polynomial is the zero one. Different kinds of objects belong to the space of ghosts, including lines, affine planes and some classes of blocking sets such as, when q is a square, Baer subplanes. Some ghosts may be seen as 2-designs (even resolvable, as in the case of Hermitian curves and hyperovals) or pairwise balanced designs, which are also resolvable or partially resolvable in some cases.

In the present talk we will find ghosts among some known substructures of PG$(2, q)$, q even, and describe their properties.

Joint work with M. Della Vedova (Chalmers University of Technology, Göteborg) and Silvia Pianta (Università Cattolica del Sacro Cuore, Brescia).

Heffter arrays with many zero-sum transversals

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Heffter arrays were introduced by Archdeacon [1] in 2015 as an interesting link between combinatorial designs and topological graph theory. A Heffter array $H(n; k)$ is an $n \times n$ matrix with entries from $\mathbb{Z}_{2nk+1}$ such that: (a) each row and each column contains k filled cells; (b) for every $x \in \mathbb{Z}_{2nk+1} \setminus \{0\}$ either x or $-x$ appears in the array; (c) the elements in every row and in every column sum to 0 in $\mathbb{Z}_{2nk+1}$. For a very recent survey on the topic see [3].

In this talk we present some results on a class of Heffter arrays (introduced and studied in [2]) satisfying much more demanding properties. These arrays are related to several well-known combinatorial objects such as magic squares, configurations and nets.

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The Oberwolfach problem with or without loving couples

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The well known Oberwolfach problem $OP(l_1, \ldots, l_t)$ asks whether $2n + 1 = l_1 + \cdots + l_t$ participants can be seated for several nights at t round tables of sizes $l_1, \ldots, l_t$ so that each participant seats next to every other exactly once. In graph-theoretic terms, the problem asks whether K_{2n+1}, the complete graph on $2n + 1$ vertices, admits a $2-$factorization such that each $2-$factor is a disjoint union of cycles of fixed lengths $l_1, \ldots, l_t$. This problem was firstly posed by Ringel in 1967 and later on some variations were considered. For example, Huang, Kotzig and Rosa (1979) proposed the analogous problem for $K_{2n} - I$, the complete graph on $2n$ vertices with a $1-$factor I removed. They called it the *the spouse avoiding variant* since it models a sitting arrangement of n couples, where each person is to sit next to each other exactly once, except that they never get to sit next to their spouse. More recently, other variations were proposed when considering complete graphs plus a repeated $1-$factor. In this talk we consider these situations and we model sitting arrangements of n couples where each person gets to sit next to exactly one other person (their partner) exactly r times and next to every other person exactly once. Our focus is particularly on solutions arising from symmetries.

On the directed Oberwolfach problem for complete symmetric equipartite digraphs

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The celebrated Oberwolfach problem, over 50 years old and in general still open, asks whether n participants at a conference can be seated at k round tables of sizes $t_1, t_2, \ldots, t_k$ for several meals so that each participant sits next to every other participant at exactly one meal, assuming that $t_1 + t_2 + \ldots + t_k = n$. This problem can be modeled as a decomposition of the complete graph K_n into 2-factors, each consisting of k disjoint cycles of lengths $t_1, t_2, \ldots, t_k$.

In this talk, we discuss the directed version for complete symmetric equipartite digraphs. Thus, we are interested in decomposing $K^*_{n[m]}$, the complete symmetric equipartite digraph with n parts of size m, into spanning subdigraphs, each a disjoint union of k directed cycles of lengths $t_1, t_2, \ldots, t_k$ (where $t_1 + t_2 + \ldots + t_k = mn$). Such a decomposition models a seating arrangement of mn participants, consisting of n delegations of m participants each, at k tables of sizes $t_1, t_2, \ldots, t_k$ so that each participant sits *to the right of each participant from a different delegation* exactly once. Recent solutions to extensive cases of this problem for uniform cycle lengths will be presented.

This is joint work with Nevena Francetić.

Approaching the minimum number of clurse Sudoku problem via the polynomial method

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In 2014 McGuire, Tugemann and Civario proved that the smallest number of clues that a Sudoku puzzle can have is 17. To do so, they performed an exhaustive computer search for 16-clue Sudoku puzzles and did not find any. They commented at the time that even a theoretical proof of the nonexistence of an 8-clue puzzle is still lacking and that a purely mathematical solution to the minimum number of clues problem is a long way off. We suggest a possible way forward using a generalization of Alon's Combinatorial Nullstellensatz. We demonstrate how this method works for the Shidoku puzzle (i.e. the 4×4 puzzle).

This is unpublished work done with a former student, Aden Forrow.

Analysis of Screening Results from Locating Arrays

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(d, t)-locating arrays have been proposed as experimental designs for screening experiments for complex systems due to their efficiency. The purpose of a screening experiment is to identify the important factors and interactions that significantly impact the measurements collected in the experiment. A (d, t)-locating array is a covering array of strength t with an additional property: Any set of d level-wise t-way interactions can be distinguished from any other such set by appearing in a distinct set of runs (rows). In this talk, we focus on the analysis of screening results, i.e., how a $(1, 2)$-locating array uses the (d, t)-locating property to recover main effects and two-way interactions from the measurements. The algorithm makes use of a compressive sensing matrix derived from the locating array, orthogonal matching pursuit, and ordinary least squares. Assumptions and limitations of the method lead to a number of challenging and novel problems in combinatorics and statistics.

Block designs obtained from square matrices

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In [1], Gunderson and Semeraro constructed a 3-$(q+1, 4, \frac{q+1}{4})$ design from a Paley tournament on q vertices, where q is a prime power with $q \equiv 3 \pmod 4$. This construction involved using blocks known as diamonds of the Paley tournament, which are tournaments on 4 vertices.

In this talk, we extend their result to Hermitian matrices that possess only a few distinct eigenvalues. This extension allows us to generate t-designs for $t = 1, 2, 3$. The matrices used as examples in this context include symmetric or skew-symmetric Hadamard matrices and Hermitian complex Hadamard matrices over the third root of unities, among others.

The content of this talk is based on joint work with Gary Greaves.

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LCD subspace codes from Hadamard matrices

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Coauthor: Dean Crnković

A subspace code is a nonempty set of subspaces of a vector space $\mathbb{F}_q^n$. Linear codes with complementary duals, or LCD codes, are linear codes whose intersection with their duals is trivial. In this talk, we will introduce a notion of LCD subspace codes. Further, we will give a construction from mutually unbiased Hadamard matrices, and more generally, from mutually unbiased weighing matrices.

Cubes of symmetric designs and difference sets

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A 3-dimensional matrix with $\{0, 1\}$ entries where each slice is an incidence matrix of a symmetric design is called a *cube of a symmetric design*. In this talk we will describe a method for a constructions of cubes of designs and establish connections with difference sets. The main question is to describes symmetric designs where each block is a difference set (not necessarily from the same development). This is joint work with Mario Osvin Pavčević and Vedran Krčadinac.

Row-sum matrices over abelian and generalized dihedral groups

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Coauthors: Andrea Burgess, Peter Danziger, Adrián Pastine

The recent concept of a *row-sum matrix*, formally introduced in [1], has proven successful in constructing factorizations of complete (equipartite) graphs [1, 2, 3] into copies of distinct uniform 2-regular graphs (that is, 2-regular graphs whose cycles have the same length).

In this talk, after emphasizing the connections with *complete mappings* [4] and *differences of bijections* [5], we present some existence results on row-sum matrices over abelian and generalized dihedral groups. The latter turn out to be very helpful in constructing uniform 2-factorizations when the cycle lengths have distinct parities.

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On resolutions of t-designs

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Generally, a t-(v, k, λ) design $(X, \mathcal{B})$ is said to have a resolution, if $(X, \mathcal{B})$ can be partitioned into mutually disjoint s-(w, k, δ) designs with $w \leq v$, $s < t$. If $w = v$, then $(X, \mathcal{B})$ is called s-resolvable; an s-resolution of the complete k-$(v, k, 1)$ design is usually called a large set of s-designs; large sets have been intensively studied, whereas resolutions of non-trivial t-designs nearly have not been investigated; several results on s-resolvable t-designs have been obtained recently, e.g. [3,4]. If $(X, \mathcal{B})$ has a resolution with $w = v - 1$, then it is called point-missing s-resolvable; in particular, a point-missing s-resolution of the complete k-$(v, k, 1)$ design is termed an overlarge set of s-designs. In this talk we mainly focus on the case $w = v - 1$ and present some new results on the subject. A remarkable feature of t-designs having resolutions is that they are very helpful for constructing t-designs, see e.g. [2]. Point-missing s-resolvable t-designs, too, prove suitable for this purpose. For example, by using known overlarge sets of Steiner quadruple systems [1], we prove the existence of various infinite series of 4-designs with constant index.

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[3] TVT, Recursive constructions for s-resolvable t-designs, *Des. Codes Cryptogr.* **87**, 2853–2845 (2019).
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Relations on nets and MOLS

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A k-net is a geometry equivalent to $(k-2)$ Mutually Orthogonal Latin Squares (MOLS). A *relation* is a linear dependence in the point-line incidence matrix of the net. In 2014 Dukes and Howard showed that any 6-net of order 10 satisfies at least two non-trivial relations. This opens up a possibile avenue towards showing the non-existence of 4 MOLS of order 10. We generated all 4-nets of order 10 that satisfy a non-trivial relation and also ruled out one type of relation on 5-nets. I will discuss these computations, as well as some of the theory of relations on nets more generally.

Higher incidence matrices and tactical decomposition matrices of designs

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In 1985, Janko and Tran Van Trung published an algorithm for constructing symmetric 2-designs with prescribed automorphisms. This algorithm is based on the equations by Dembowski (1958) for tactical decompositions of point-block incidence matrices. In the sequel, the algorithm has been generalized and improved in many articles.

In parallel, higher incidence matrices for t-designs with arbitrary strength $t \geq 2$ have been introduced by Wilson in 1982. They have proven useful for obtaining several restrictions on the existence of designs. For example, a short proof of the generalized Fisher's inequality makes use of these incidence matrices.

In this talk we present a unified approach to tactical decompositions and incidence matrices. It works for both combinatorial and subspace designs alike. As a result, we obtain a generalized Fisher's inequality for tactical decompositions of combinatorial and subspace designs. Moreover, our approach is explored for the construction of combinatorial and subspace designs of arbitrary strength.

This is joint work with Michael Kiermaier, University of Bayreuth.

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Minisymposium

Configurations

Organized by Gábor Gévay, *University of Szeged*

INVITED TALK

From Zindler to Berman

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We start with a short history of the mathematics of Konrad Zindler (1866-1934), whose Liniengeometrie mit Anwendungen is now again available through on-demand-printing. His generalized configurations are non-rigid by construction. Leah Berman's movable configurations are of quite different nature and lead to the Projective Rigidity project involving Leah Berman, Klara Stokes, Walter Whitely and B.H. Servatius.

Configurations and the simple Lie algebras

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I will be discussing a connection of configurations and simple Lie algebras which we observed during our investigation of the universal dimension formulae of the latter.

In the framework of the universal approach (proposed by Vogel, Deligne, et al.), the parametrization of simple Lie algebras is based on the points in the projective plane P^2. Notably, certain important quantities of the simple Lie algebras are expressed as specific rational functions of homogeneous coordinates of P^2.

Our focus is on the issue of uniqueness regarding these functions. We have discovered that this problem is equivalent to the existence of specific configurations of points and lines. In particular, we have found that valid configurations must be "colorable" in a specific manner.

I will explain how the colorable versions of (9_3) and $(16_3, 12_4)$ configurations have partially resolved the uniqueness problem.

Additionally, I will discuss a colorable $(144_3, 36_{12})$ configuration that has the potential to completely solve the problem.

Toric manifolds and colorings of discrete structures

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Small covers and quasitoric manifolds are topological generalizations of smooth projective toric varieties. Their topological features are determined by combinatorial polytopes that are the orbit space of the corresponding actions of a real and a complex torus. Therefore, many important problems in toric topology can be reduced to problems of generalized colourings of discrete structures such as graphs and simplicial complexes. This talk presents some of them and explains deep correlations among combinatorics, algebra and topology.

New results on (21_4) configurations

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Coauthors: Gábor Gévay, Tomaž Pisanski

A (n_k) geometric configuration is a collection of points and straight lines, typically in the Euclidean plane, in which every point lies on k lines and every line passes through k points. The modern study of configurations was initiated with the 1990 publication by Branko Grünbaum and John Rigby of the first geometric realization of an (n_4) configuration, a (21_4) configuration (now called the Grünbaum-Rigby configuration) whose points form the vertices of three concentric heptagons. In this talk, we discuss recent results about (21_4) configurations, including a construction of a new polycyclic (21_4) configuration, which is the second known such configuration, and new geometric realizations of the Grünbaum-Rigby configuration.

Self-inscribable configurations

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A balanced configuration $\mathcal{C}$ is called *self-inscribable* if there is a suitably rotated homothetic copy $\mathcal{C}'$ such that the points of $\mathcal{C}'$ are incident to the lines of $\mathcal{C}$. When $\mathcal{C}'$ is positioned so, we say that it is *inscribed* in $\mathcal{C}$. In this way new configurations can be constructed such that they are composed of cyclically inscribed copies of a starting polycyclic configuration of a well-determined simple structure. Taking an (n_k) configuration, the inscribing procedure can be repeated arbitrary often, so that the incidence number k increases by one at each step; hence these constructions provide configurations with arbitrary large incidence number. In my talk I present infinite families and sporadic examples of such new configurations; some of the latter are of special interest. To mention two of them here: one is a (84_5) example composed of 4 cyclically insribed copies of the renowned (21_4) Grünbaum–Rigby configuration; the other is a (60_5) example which is the smallest currently known movable 5-configuration.

Disclosing Quantum Contextuality: A Geometric Approach to N-Qubit Configurations

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Coauthors: Metod Saniga, Alain Giorgetti, Henri de Boutray, Frédéric Holweck

In the intriguing realm of quantum physics, the concept of contextuality raises interesting questions about the nature of quantum systems. Since the commutation relation in the N-qubits Pauli group corresponds to the colinearity relation in the symplectic polar space $W(2N-1,2)$, we study subgeometries of this space in order to discover new contextual quantum configurations.

We have developed algorithms and a corresponding software in C language, that can effectively determine quantum contextuality and quantify it, by charting these symplectic polar spaces of various ranks.

This tool allowed us to uncover new insights that go beyond the achievements of earlier work, such as that of de Boutray et al (J. Phys. A: Math. Theor. 55 475301, 2022). Our study spans across these geometric structures of ranks from two to seven, offering significant findings.

In my talk I will explicitly illustrate the concept of quantum contextuality on certain well-known finite-geometric configurations living in W(5,2), like the doily (and its two-spread), an elliptic quadric (and its spread of lines) and a hyperbolic quadric (and its associated Heawood graph); I will also consider configurations whose contexts are all the subspaces with a given dimension k < N.

On Polycirculant Graphs and Polycyclic Configurations

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Polycirculant graphs constitute a natural generalisation of circulant graphs. In particular, an m-circulant (also called an m-multicirculant) is a cyclic cover over the base graph of order m. Several important graph families such as generalised Petersen graphs, rose-window graphs, etc. belong to the class of polycirculants. In this talk, we will review some properties of polycirculant graphs, such as symmetry, hamiltonicity, spectral properties, etc. that have been studied intensively in this millennium. In particular, we will consider small cases of $m, 1 \leq m \leq 4$, i.e. circulants, bicirculants, tricirculants and tetracirculants. Polycirculants are closely related to polycyclic configurations [3, 4], introduced in [1]. Namely, the Levi graph [2] of a polycyclic configuration is a bipartite polycirculant graph of girth at least six. On the one hand, the covering projection of Levi graph to the base graph, known as reduced Levi graph admits concise description of the corresponding configuration, on the other hand, by changing the voltage group, an infinite series of configurations having several common properties is obtained. In addition, several properties of graphs, such as splittability and dimension have been motivated by the study of configurations. The talk is based on the joint work with several colleagues.

This work is supported in part by the Slovenian Research Agency (research program P1-0294 and research projects J1-1690, N1-0140, J1-2481).

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Computer generation of incidence theorems

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I will describe (and run) an algorithm that constructs configurations that exhibit interesting theorems in projective geometry. I will explain what the first few configurations represent, and show others that seem interesting but are still unexplained.

Contextual configurations

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Coauthors: Adan Cabello, Petr Lisonek

Contextuality, a phenomenon predicted by quantum physics, has recently been identified as a key resource for quantum computing. One particular structure which captures this phenomenon is given by an even-degree hypergraph labeled with operators from the n-qubit Pauli group satisfying certain requirements. The two most well-known such structures are the 2-qubit Mermin Square, a $(9_2, 6_3)$ configuration, and the 3-qubit Peres-Mermin Pentagram, a $(10_2, 5_4)$ configuration. A result of Arkhipov (2012) shows that if a contextual hypergraph is 2-regular, then it admits a labeling with 2-qubit or 3-qubit Pauli operators by reducing to the Square or Pentagram. Prior to our work, all contextual hypergraphs known could be reduced to one of these cases.

We present an efficient algorithm that checks whether a hypergraph admits a contextual labeling, and if so it generates a labeling with the minimum number of qubits. By considering contextual hypergraphs with vertices of degree greater than two, we found examples that cannot be reduced to the Square or Pentagram. Of these, there is the (21_4) Grünbaum-Rigby configuration (which requires 4 qubits), a 3-astral 4-configuration (which requires 4 qubits), the smallest known weakly flag-transitive configuration (which requires 5 qubits), as well as a (27_4) configuration (which requires 3 qubits). Our hope is to find an infinite family of contextual configurations that cannot be reduced.

Joint work with Adán Cabello (Universidad de Sevilla) and Petr Lisoněk (Simon Fraser University).

Minisymposium

Graph Colorings

Organized by Théo Pierron, *LIRIS, Université Claude Bernard Lyon 1*
Coorganized by Jonathan Narboni, *Jagiellonian University*

INVITED TALK
The list-packing number

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Coauthors: Stijn Cambie, Ewan Davies, Ross Kang

We are interested in a natural strengthening of list-colouring, where instead of one list-colouring we seek many in parallel. More precisely, given the assignment of a list $L(v)$ of k colours to each vertex $v \in V(G)$, we study the existence of k pairwise-disjoint proper colourings of G using colours from these lists. We refer to this as a *list-packing* and we define the *list-packing number* $\chi_\ell^\star(G)$ as the smallest k for which every list-assignment of G admits a list-packing.

We prove several results that (asymptotically) match the best-known bounds for the list chromatic number, among which: $\chi_\ell^\star(G) \leq n$ with equality if and only if G is the complete graph on n vertices, $\chi_\ell^\star(G) \leq (1+o(1))\log_2(n)$ if G is bipartite on n vertices, and $\chi_\ell^\star(G) \leq (1+o(1))\Delta/\log(\Delta)$ if G is bipartite with maximum degree Δ. However, in general the condition that colourings are disjoint seems to make the problem less susceptible to local arguments, so that greedy bounds for the (list) chromatic number do not automatically carry over to the list-packing setting. To illustrate this: a greedy argument only produces $\chi_\ell^\star(G) \leq 2\Delta$, for every graph G of maximum degree Δ, while we conjecture that in fact $\chi_\ell^\star(G) \leq \Delta + 1$. The latter is true for $\Delta \leq 3$, and we als prove a fractional analogue for all Δ. In contrast, Brook's Theorem cannot carry over without modification; for instance, there exists a connected 3-regular graph, not a clique, which has (fractional) list-packing number 4.

There is a plethora of questions left. What about the optimal bound for planar graphs, line graphs, random graphs, bounded treewidth ...? Our main open question is whether $\chi_\ell^\star(G)$ can be bounded by a uniform constant times the list chromatic number.

Based on joint works with Stijn Cambie, Ewan Davies and Ross Kang.

Coloring zonotopal quadrangulations on the projective space

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A *quadrangulation* on a surface is a fixed embedding of a graph on the surface such that every face is quadrilateral. It is easy to see that every quadrangulation on the sphere is bipartite, but every non-spherical surface admits a non-bipartite quadrangulation. In particular, Young pointed out a strange fact that every quadrangulation on the projective plane is either bipartite or 4-chromatic, though it is known that every even-sided map on any non-spherical orientable surface with high width is 3-colorable. In our talk, we discuss a d-dimensional analogue of the Young's theorem for any integer $d \geq 2$, and consider a relation to another result on high-dimensional quadrangulations on the projective space by Kaiser and Stehlíc.

Injective coloring of graphs revisited

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An open packing in a graph G is a set S of vertices in G such that no two vertices in S have a common neighbor in G. The injective chromatic number $\chi_i(G)$ of G is the smallest number of colors assigned to vertices of G such that each color class is an open packing. Alternatively, the injective chromatic number of G is the chromatic number of the two-step graph of G, which is the graph with the same vertex set as G in which two vertices are adjacent if they have a common neighbor. Injective colorings have been investigated by many authors, while in this talk we present two novel approaches, related to open packings and the two-step graph operation. In particular, we present several bounds on the injective chromatic number of a graph involving its order, size and the open packing number. We also consider the chromatic number of the two-step graph of a graph, and compare it with the clique number and the maximum degree of the graph. Finally, we consider classes of graphs that admit an injective coloring in which all color classes are maximal open packings.

Unique-maximum colorings of plane graphs

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A proper vertex coloring of a plane graph is unique-maximum (UM) if, for every face, the maximum color on its vertices is used exactly once. Wendland (2016) proved that every plane graph is UM 5-vertex-colorable and Lidický, Messerschmidt, and Škrekovski (2018) constructed a plane graph with the corresponding chromatic number 5.

In this talk, we consider the following two modifications of the described coloring:

(1) a proper vertex-face coloring of a plane graph is unique-maximum (UM) if, for every face, the maximum color on its vertices and on the face itself is used exactly once;

(2) a proper vertex coloring of a plane graph is unique-double-maximum (U2M) if, for every face, the highest color and the second highest color on its vertices are used exactly once;

We compare these two colorings and present upper bounds on the corresponding chromatic numbers for the set of plane graphs.

Majority distinguishing edge colorings

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An edge coloring c of a graph G is called a *majority edge coloring* if for every vertex v of the graph G and every color α at most half of the edges incident with v have the color α. We are interested in finding the smallest possible number of colors in such a coloring. Bock et al.

proved that every graph of minimum degree at least two has a majority edge coloring with four colors and this bound is best possible in general. The *distinguishing index* $D'(G)$ of a graph G is the least number d such that G has an edge coloring with d colors that is only preserved by the identity automorphism. Imrich et al. proved that if G is a connected graph with minimum degree at least two, then $D'(G) \leq \lceil \Delta(G) \rceil + 1$.

In the talk we present upper bounds on the number of colors in an edge coloring of a graph that is both a majority coloring and a distinguishing coloring. We show a general bound for all graphs of degree at least two and some results for selected classes of graphs. In particular, we show that if G is a traceable graph of minimum degree at least four, then G has a majority distinguishing edge coloring with three colors.

A generalization of properly colored paths and cycles in edge-colored graphs

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A walk in an edge-colored graph is said to be a properly colored walk iff every two consecutive edges have different color, this includes the first and last edges when the walk is closed. Properly colored walks have shown to be an effective way to model certain real applications in different fields. In view of this, it is natural to ask about the existence of properly colored walks with restrictions in the transitions of colors allowed in the edges of a graph.
Let H be a graph possibly with loops and G a graph. We say that G is an H-colored graph whenever there exists a function $c : E(G) \longrightarrow V(H)$. A path $(v_1, \cdots, v_n)$ in an H-colored graph G is an H-path iff $(c(v_1v_2), c(v_2v_3), \cdots, c(v_{n-1}v_n))$ is a walk in H, and a cycle $(v_1, \cdots, v_n, v_1)$ is an H-cycle whenever $(c(v_1v_2), c(v_2v_3), \cdots, c(v_{n-1}v_n), c(v_nv_1), c(v_1v_2))$ is a walk in H. Hence, H decides which color transitions are allowed in a walk in order to be an H-walk. In this talk, we show conditions implying the existence of H-paths and H-cycles of certain length in an H-colored graph with a given structure.

Majority Edge-Colorings of Graphs

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A coloring (labeling) of edges of a graph G is called majority, if for every vertex u of G, at most half the edges incident with u have the same color. This concept was introduced in [1], where we proved that every finite graph without pendant vertices admits a majority 4-edge coloring. Moreover, if minimum degree of G is at least 4, then G admits a majority 3-edge coloring.

In the talk, infinite graphs will be investigated.

References:

[1] F. Bock, R. Kalinowski, J. Pardey, M. Pilśniak, D. Rautenbach, M. Woźniak, Majority Edge-Colorings of Graphs, Electron. J. Combin. 30 (2023) P1.42; doi:10.37236/11291

A Classification for the Complexity of the Matroid-Homomorphism Problems

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We introduce the difference between graph-homomorphisms and matroid-homomorphisms. With this we prove a complexity dichotomy for the problem $\mathrm{Hom}_{\mathbb{M}}(\mathrm{N})$ of deciding if there is a matroid-homomorphism to N. The problem is polynomial time solvable if N has a loop or has no circuits of odd length, and is otherwise NP-complete. This is joint work with Mark Siggers at Kyungpook National University and Cheolwon Heo at KIAS.

The square of every subcubic planar graph of girth at least 6 is 7-choosable

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The square of a graph G, denoted G^2, has the same vertex set as G and has an edge between two vertices if the distance between them in G is at most 2. Wegner's conjecture (1977) states that for a planar graph G, the chromatic number $\chi(G^2)$ of G^2 is at most 7 if $\Delta(G) = 3$, at most $\Delta(G)+5$ if $4 \leq \Delta(G) \leq 7$, and at most $\lfloor \frac{3\Delta(G)}{2} \rfloor$ if $\Delta(G) \geq 8$. Wegner's conjecture is still wide open. The only case for which we know tight bound is when $\Delta(G) = 3$. Thomassen (2018) showed that $\chi(G^2) \leq 7$ if G is a planar graph with $\Delta(G) = 3$, which implies that Wegner's conjecture is true for $\Delta(G) = 3$. A natural question is whether $\chi_\ell(G^2) \leq 7$ or not if G is a subcubic planar graph, where $\chi_\ell(G^2)$ is the list chromatic number of G^2. Cranston and Kim (2008) showed that $\chi_\ell(G^2) \leq 7$ if G is a subcubic planar graph of girth at least 7. We prove that $\chi_\ell(G^2) \leq 7$ if G is a subcubic planar graph of girth at least 6. This is joint work with Xiaopan Lian.

Uniquely colorable graphs up to automorphisms

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We extend the concept of uniquely colorable graphs and say that a graph G is χ-iso-unique if for every two proper colorings $c : V(G) \to \{1, \ldots, \chi(G)\}$ and $d : V(G) \to \{1, \ldots, \chi(G)\}$ there exists an automorphism φ of G such that for any $i \in \{1, \ldots, \chi(G)\}$ there exists $j \in \{1, \ldots, \chi(G)\}$ so that φ maps $c^{-1}(i)$ onto $d^{-1}(j)$.

We show some general properties of degrees of vertices of χ-iso-unique graph G, and then focus on non-bipartite outerplanar χ-iso-unique graphs. As proved by Chartrand and Geller back in 1969, uniquely 3-colorable outerplanar graphs are precisely the 2-connected outerplanar graphs in which all induced cycles are triangles. We show that, in contrast to uniquely 3-colorable graphs, the χ-iso-unique outerplanar graphs with $\chi(G) = 3$ may have (at most) one inner face which is not a triangle, and it is isomorphic to C_5 if it exists. Moreover, the χ-iso-unique graphs may have cut-vertices.

We characterize all outerplanar χ-iso-unique graphs and show that χ-iso-unique outerplanar graphs can be recognized efficiently.

Brooks-type theorems for relaxations of square coloring

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The following relaxation of a proper coloring of the square of a graph was recently introduced: for a positive integer h, the proper h-conflict-free k-coloring of a graph G is a proper k-coloring of G such that for every vertex v has $\min\{\deg_G(v), h\}$ colors uniquely appearing on its neighborhood. The proper h-conflict-free chromatic number of a graph G, denoted $\chi^h_{pcf}(G)$, is the minimum k such that G has a proper h-conflict-free k-coloring. A Brooks-type conjecture was proposed by Caro, Petruševski, and Škrekovski, and its content is as follows: if G is a graph with $\Delta(G) \geq 3$, then $\chi^1_{pcf}(G) \leq \Delta(G) + 1$. Regarding the conjecture, Pach and Tardos proved $\chi^1_{pcf}(G) \leq 2\Delta(G) + 1$. We improve the result for all h: if G is a graph with $\Delta(G) \geq h + 2$, then $\chi^h_{pcf}(G) \leq (h+1)\Delta(G) - 1$. Also, we show that the conjecture is true for chordal graphs, and obtain partial results for quasi-line graphs and claw-free graphs.

Sets of r-graphs that color all r-graphs

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An r-regular graph is an r-graph, if every odd set of vertices is connected to its complement by at least r edges. Let G and H be r-graphs. An *H-coloring* of G is a mapping $f\colon E(G) \to E(H)$ such that each r adjacent edges of G are mapped to r adjacent edges of H. For every $r \geq 3$, let $\mathcal{H}_r$ be an inclusion-wise minimal set of connected r-graphs, such that for every connected r-graph G there is an $H \in \mathcal{H}_r$ which colors G.

In this talk, we show that $\mathcal{H}_r$ is unique and characterize $\mathcal{H}_r$ by showing that $G \in \mathcal{H}_r$ if and only if the only connected r-graph coloring G is G itself.

The Petersen Coloring Conjecture states that the Petersen graph P colors every bridgeless cubic graph. We show that if true, this is a very exclusive situation. Indeed, either $\mathcal{H}_3 = \{P\}$ or $\mathcal{H}_3$ is an infinite set and if $r \geq 4$, then $\mathcal{H}_r$ is an infinite set. Similar results hold for the restriction on simple r-graphs. Moreover, we determine the set of smallest r-graphs of class 2 and show that it is a subset of $\mathcal{H}_r$.

This is joint work with Davide Mattiolo, Eckhard Steffen, and Isaak H. Wolf.

3-COLORING in time $\mathcal{O}^*(1.3217^n)$.

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We propose a new algorithm for 3-COLORING that runs in time $\mathcal{O}^*(1.3217^n)$. For this algorithm, we make use of the time $\mathcal{O}^*(1.3289^n)$ algorithm for 3-COLORING by Beigel and Eppstein. They described a structure in all graphs, whose vertices could be colored relatively easily. In this paper, we improve upon this structure and present new ways to determine how the involved vertices reduce the runtime of the algorithm.

On acyclic b-chromatic number of a graph

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Let G be a graph. We introduce acyclic b-chromatic number of G as an analog to b-chromatic number of G. An acyclic coloring of a graph G is a map $c : V(G) \to \{1, \ldots, k\}$ such that $c(u) \neq c(v)$ for any $uv \in E(G)$ and the induced subgraph on vertices of any two colors $i, j \in \{1, \ldots, k\}$ induce a forest. On a set of all acyclic colorings of a graph G we define a relation whose transitive closure is a strict partial order. The minimum cardinality of its minimal element is then the acyclic chromatic number $A(G)$ of G and the maximum cardinality of its minimal element is the acyclic b-chromatic number $A_b(G)$ of G. We present several properties of $A_b(G)$. In particular, we derive $A_b(G)$ for several known graph families, we compare the behavior of acyclic b-chromatic number with respect to b-chromatic number on cubic graphs, derive some bounds for $A_b(G)$, compare $A_b(G)$ with some other parameters and generalize some influential tools from b-colorings to acyclic b-colorings. The most surprising, even anti-intuitive, is that acyclic b-chromatic number of a graph can be arbitrary smaller than b-chromatic number.

Uniformly random colourings of sparse graphs

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Given a random Δ-regular graph G, it holds that $\chi(G) \sim \frac{\Delta}{2\ln\Delta}$ with high probability. However, for any $\varepsilon > 0$ and k large enough, no (randomised) polynomial-time algorithm returning a proper k-colouring of such a random graph G is known to exist when $k < (1-\varepsilon)\frac{\Delta}{\ln\Delta}$, while a greedy algorithm can return a proper k-colouring when $k > (1+\varepsilon)\frac{\Delta}{\ln\Delta}$. One of the reasons that could explain this is that the space of proper k-colourings of a random graph is composed of many far-apart clusters when $k < (1-\varepsilon)\frac{\Delta}{\ln\Delta}$, whence the small probability of success of a local search algorithm to find such a colouring. In contrast, when $k > (1+\varepsilon)\frac{\Delta}{\ln\Delta}$, the space of proper k-colourings is well-connected in many aspects. In that setting, given a random Δ-regular graph G and a random k-colouring of G, with high probability one can recolour any vertex of G with any colour by recolouring only a sublinear fraction of the vertices.

With respect to proper colourings, sparse graphs share many extremal properties with random graphs. In 2019, Molloy used entropy compression (an algorithmic version of the Lovasz Local Lemma introduced in 2010 by Moser and Tardos) to construct a randomised polynomial-time algorithm that yields a proper k-colouring of any triangle-free G of maximum degree Δ, with $k = (1 + o(1))\frac{\Delta}{\ln \Delta}$ as $\Delta \to \infty$. In this presentation, we will show that, with high probability, in a uniformly random proper k-colouring of G all but a small fraction of the vertices can be recoloured to any colour by recolouring only their neighbours. The same holds with high probability for every vertex if we require that the girth of G is large enough.

On generalized majority colourings

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A *majority colouring* of a directed graph, first studied by Kreutzer, Oum, Seymour, van der Zypen, and Wood, is a vertex colouring in which each vertex has the same colour as at most half of its out-neighbors. It is conjectured that every directed graph is majority 3-colourable. Within the talk we shall discuss a few known results concerning this and related concepts, and present how to simplify some proof techniques and generalize previously known results on various generalizations of majority colouring. In particular, our unified and simplified approach works for *paintability* – an on-line analog of the list colouring.

3-colourability and diamonds

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The 3-colourability problem is an NP-complete problem which remains NP-complete for claw-free graphs and for graphs with maximum degree four. In this talk we will consider induced subgraphs, among them are the *claw* ($K_{1,3}$), the *bull* (a triangle with two pendent edges), and the *diamond* (the graph $K_4 - e$.

Our main result is a complete characterization of all 3-colourable $(claw, bull)$-free graphs. We will present a description of all non 3-colourable $(claw, bull)$-free graphs in terms of diamonds. Moreover, we will show extensions of this characterization to larger graph classes by taking supergraphs of the claw or the bull.

Locally irregular edge-coloring of subcubic graphs

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A graph is *locally irregular* if no two adjacent vertices have the same degree. A *locally irregular edge-coloring* of a graph G is such an (improper) edge-coloring that the edges of any fixed color induce a locally irregular subgraph. Among the graphs admitting a locally irregular edge-coloring, i.e., *decomposable graphs*, only one is known to require 4 colors, while for all the others it is believed that 3 colors suffice. In the talk, we prove that decomposable claw-free graphs with maximum degree 3, all cycle permutation graphs, and all generalized Petersen graphs admit a locally irregular edge-coloring with at most 3 colors. We also discuss when 2 colors suffice for a locally irregular edge-coloring of cubic graphs and present an infinite family of cubic graphs of girth 4 which require 3 colors. Finally, we will also deal with powers of cycles and show that these $2k$-regular graphs admit locally irregular edge-colorings with 2 colors except for the case of complete graphs.

Cyclic colorings of plane graphs and its relaxations

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A cyclic coloring of plane graphs is a vertex coloring in which all vertices incident with the same face receive distinct colors. The Cyclic Coloring Conjecture states that every plane graph with maximum face size Δ^* admits a cyclic coloring with at most $\lfloor 3\Delta^*/2 \rfloor$ colors. One possible relaxation of cyclic coloring is the ℓ-facial coloring of plane graphs where vertices at facial distance at most ℓ receive distinct colors. An open conjecture states that every plane graph admits an ℓ-facial coloring with $3\ell + 1$ colors. Another possible relaxation of cyclic coloring is a facial-parity coloring (known also as a strong odd coloring) of plane graphs which is a vertex coloring in which on the boundary of every face each color appears zero or an odd number of times. It was believed that 10 colors would always suffice for such a coloring. In a natural way also the edge variants of the above three colorings are defined.

In this talk we give an overview on the developments of the above mentioned facial colorings of plane graphs, as well as our recently published results. For $\ell = 3$ in the case of the edge variant of the ℓ-facial coloring we prove that 10 colors are always enough. As for the facial-parity coloring we provide examples of plane graphs requiring 12 colors for both the edge and vertex variants of the coloring.

Some results on the palette index of graphs

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Given an edge-coloring of a graph, the palette of a vertex is defined as the set of colors of the edges which are incident with it. We define the palette index of a graph as the minimum number of distinct palettes, taken over all edge-colorings, occurring among the vertices of the graph. Several results about the palette index of some specific classes of graphs are known. In this talk we present some new results on the palette index of graphs. Our main theorem gives a sufficient condition for a graph to have palette index larger than its minimum degree. We also give a characterization of graphs with “small" palette index (either 2 or 3) in terms of the existence of suitable decompositions in regular subgraphs. Using previous results, in the talk we present, for every odd r, a family of r-regular graphs with palette index reaching the maximum admissible value, the first known family of simple graphs whose palette index grows quadratically with respect to their maximum degree and we finally provide a complete characterization of regular graphs having palette index 3.

Minisymposium

Graph Domination

Organized by Michael Henning, *University of Johannesburg, South Africa*
Coorganized by Csilla Bujtas, *University of Ljubljana*

INVITED TALK

The $\frac{1}{3}$-Conjectures for Domination in Cubic Graphs

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A set S of vertices in a graph G is a dominating set of G if every vertex not in S is adjacent to a vertex in S. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set in G. In a breakthrough paper in 2008, Löwenstein and Rautenbach [Graphs Combin. 24(1) (2008), 37–46] proved that if G is a cubic graph of order n and girth at least 83, then $\gamma(G) \leq \frac{1}{3}n$. A natural question is if this girth condition can be lowered. The question gave birth to two $\frac{1}{3}$-conjectures for domination in cubic graphs. The first conjecture, posed by Verstraete in 2010, states that if G is a cubic graph on n vertices with girth at least 6, then $\gamma(G) \leq \frac{1}{3}n$. The second conjecture, first posed as a question by Kostochka in 2009, states that if G is a cubic, bipartite graph of order n, then $\gamma(G) \leq \frac{1}{3}n$. In this talk, we prove Verstraete's conjecture when there is no 7-cycle and no 8-cycle, and we prove the Kostochka's related conjecture for bipartite graphs when there is no 4-cycle and no 8-cycle.

General sharp upper bounds on the total coalition number

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Let $G(V, E)$ be a finite, simple, isolate-free graph. Two disjoint sets $A, B \subset V$ form a total coalition in G, if none of them is a total dominating set, but their union $A \cup B$ is a total dominating set. A vertex partition $\Psi = \{C_1, C_2, \ldots, C_k\}$ is a total coalition partition, if none of the partition classes is a total dominating set, meanwhile for every $i \in \{1, 2, \ldots, k\}$ there exists a distinct $j \in \{1, 2, \ldots, k\}$ such that C_i and C_j form a total coalition. The maximum cardinality of a total coalition partition of G is the total coalition number of G and denoted by $TC(G)$. We give a general sharp upper bound on the total coalition number as a function of the maximum degree. We further investigate this optimal case and study the total coalition graph. We show that every graph can be realised as a total coalition graph.

4-Total Domination Game Critical Graphs

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The total domination game is played on a simple graph G with no isolated vertices by two players, Dominator and Staller, who alternate choosing a vertex in G. Each chosen vertex totally dominates its neighbors. In this game, each chosen vertex must totally dominates at least one new vertex not totally dominated before. The game ends when all vertices in G are totally dominated. Dominator's goal is to finish the game as soon as possible, and Staller's goal is to prolong it as much as possible. The game total domination number is the number of chosen vertices when both players play optimally, denoted by $\gamma_{tg}(G)$ when Dominator starts the game and denoted by $\gamma'_{tg}(G)$ when Staller starts the game. If a vertex v in G is declared to be already totally dominated, then we denote this graph by $G|v$. A total domination game critical graph is a graph G for which $\gamma_{tg}(G|v) < \gamma_{tg}(G)$ holds for every vertex v in G. If $\gamma_{tg}(G) = k$, then G is called k-γ_{tg}-critical. In this work, we characterize some 4-γ_{tg}-critical graphs.

Hamilton Paths in Dominating Graphs

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The dominating graph of a graph G has as its vertices all dominating sets of G, with two vertices adjacent if one of the corresponding dominating sets can be obtained from the other by the addition or deletion of a single vertex of G. We are interested in the properties of such graphs. In particular, we show that the dominating graph of any tree has a Hamilton path and that the dominating graph of a cycle on n vertices has a Hamilton path if and only if $n \not\equiv 0 \pmod 4$.

Maker-Breaker domination game on subdivided stars

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In the Maker-Breaker domination game played on a graph G, two players; Dominator and Staller; alternately select an unplayed vertex in G. Dominator's goal is to form a dominating set and Staller's goal is to claim a closed neighborhood of some vertex. We study the cases when Staller can win the game. If Dominator (resp., Staller) starts the game, then $\gamma_{\mathrm{SMB}}(G)$ (resp., $\gamma'_{\mathrm{SMB}}(G)$) denotes the minimum number of moves Staller needs to win. Let $S = S(n_1, \ldots, n_\ell)$ be the subdivided star obtained from the star with ℓ edges by subdividing its edges $n_1 - 1, \ldots, n_\ell - 1$ times, respectively. Then $\gamma'_{\mathrm{SMB}}(S)$ is determined in all the cases except when $\ell \geq 4$ and each n_i is even. The simplest formula is obtained when there are at least two odd n_is. If n_1 and n_2 are the two smallest such numbers, then $\gamma'_{\mathrm{SMB}}(S(n_1, \ldots, n_\ell)) = \lceil \log_2(n_1 + n_2 + 1) \rceil$.

Optimal linear-Vizing relationships for (total) domination

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Building on a classical result of Vizing relating the size, order and domination number of a graph, in 1999 Rautenbach discovered that the quadratic relation implied by Vizing's theorem could be improved to a linear relation in n, γ, and bounded maximum degree Δ. This led to the question of finding the optimal form of such inequality: What is the smallest c_Δ such that

every connected n vertex graph with m edges, domination number γ, and maximum degree Δ has $m \leq \binom{\Delta+c_\Delta}{2} - \binom{\Delta+c_\Delta+2}{2}\gamma$. Rautenbach's original work showed $c_\Delta \leq \Delta$. A similar linear-Vizing result was developed for total domination, and the corresponding constant r_Δ was shown to satisfy $0.1 \ln \Delta < r_\Delta \leq 2\sqrt{\Delta}$ by Yeo. Yeo left as an open question '... whether r_Δ grows proportionally to $\ln \Delta$ or $\sqrt{\Delta}$ or some completely different function.'

In this talk, we establish the asymptotics of both r_Δ and c_Δ, showing that both are asymptotically $\ln(\Delta)$. This is based on joint work with Mike Henning.

Cops & Robbers Pebbling in Graphs

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Here we merge the two fields of Cops and Robbers and Graph Pebbling to introduce the new topic of Cops and Robbers Pebbling. Both paradigms can be described by moving tokens (the cops) along the edges of a graph to capture a special token (the robber). In Cops and Robbers, all tokens move freely, whereas, in Graph Pebbling, some of the chasing tokens disappear with movement while the robber is stationary. In Cops and Robbers Pebbling, some of the chasing tokens (cops) disappear with movement, while the robber moves freely. We define the cop pebbling number of a graph to be the minimum number of cops necessary to capture the robber in this context, and present upper and lower bounds and exact values, some involving various domination parameters, for an array of graph classes. We also offer several interesting problems and conjectures.

Domination of subcubic planar graphs with large girth

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Since in 1996 Reed conjectured that the domination number of a connected cubic graph of order n is at most $\lceil \frac{1}{3}n \rceil$, the domination number of cubic graphs has been extensively studied. It is now known that the conjecture is false in general, but that it holds for graphs with girth at least 83. An analogous conjecture has also been stated for connected cubic planar graphs.

In this talk we present a new upper bound for the domination number of subcubic planar graphs: if G is a subcubic planar graph with girth at least 8, then $\gamma(G) \leq \lfloor n_0 + \frac{3}{4}n_1 + \frac{11}{20}n_2 + \frac{7}{20}n_3 \rfloor$, where n_i denotes the number of vertices of degree i in G for $i \in \{0, 1, 2, 3\}$.

Domination Parameters on Cayley Digraphs of Full Transformation Semigroups Relative to Green's Equivalence $\mathcal{L}$-classes and Their Complements

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Let $T(X)$ be a full transformation semigroup on a nonempty set X. The Cayley digraph Γ of a semigroup $T(X)$ with respect to the Green's $\mathcal{L}$-class is a digraph with vertex set $T(X)$ and two vertices $\alpha, \beta \in T(X)$ are adjacent as an arc (α, β) whenever $\beta = \alpha\mu$ for some μ in a connection set $\mathcal{L}$-class. In this talk, we present the domination parameters of Cayley digraphs Γ and their complements $\overline{\Gamma}$ consisting of domination, total domination, independent domination, connected domination, and split domination.

Orientable domination in product-like graphs

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A set S of vertices in a finite digraph D is a dominating set of D if for every vertex x not in S there exists an arc (s, x) in D for some $s \in S$. The *domination number* of D is the minimum cardinality of a dominating set of D.

The *orientable domination number* of a finite simple graph G, denoted by $\mathrm{DOM}(G)$, is the largest domination number taken over all orientations of G. The first publication on the orientable domination number (using different language) was by Erdös in 1963. He established lower and upper bounds for $\mathrm{DOM}(T_n)$ where T_n is a tournament of order n. This concept was first defined and studied for arbitrary graphs by Chartrand, VanderJagt and Yue in 1996.

In this paper, DOM is studied on different product graphs and related graph operations. The orientable domination number of arbitrary corona products is determined, while sharp lower and upper bounds are proved for Cartesian and lexicographic products. A result of Chartrand et al. from 1996 is extended by establishing the values of $\mathrm{DOM}(K_{n_1,n_2,n_3})$ for arbitrary positive integers n_1, n_2 and n_3.

Minisymposium

Metric Dimension And Related Topics

Organized by Ismael Yero, *Cadiz University*

INVITED TALK

On some recent contributions on the metric dimension of graphs with some open problems

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Given a graph G and a subset of vertices $S = \{w_1, \ldots, w_t\} \subseteq V(G)$, the metric representation of a vertex $u \in V(G)$ with respect to S is the vector $r(u|S) = (d_G(u, w_1), \ldots, d_G(u, w_t))$, where $d_G(x, y)$ represents the length of a shortest x, y-path in G. A subset of vertices S such that $r(u|S) = r(v|S)$ if and only if $u = v$ for every $u, v \in V(G)$ is said to be a resolving set for G, and the cardinality of a smallest such set is the metric dimension of G. Several variations of the metric dimension of graphs are known nowadays.

A few interesting results on the metric dimension of graphs and on some of its variants shall be given in this work. In addition, a few remarkable and recent open problems in the topic will be commented.

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The Threshold Strong Dimension of Trees

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Let G be a graph and W be a set of vertices of G. A vertex w in W is said to strongly resolve two vertices u and v in G if there is either a shortest u-w path that contains v or a shortest v-w path that contains u. The set W is called a strong resolving set if every pair of vertices in G is strongly resolved by a vertex of W. A smallest strong resolving set is called a strong basis and its cardinality, the strong dimension, denoted $\beta_s(G)$. When additional edges are added to a graph G, the strong dimension $\beta_s(G)$ can either increase, decrease, or remain the same. This observation lead to the introduction of a new parameter called the threshold strong dimension of G, denoted $\tau_s(G)$. It represents the smallest strong dimension among all graphs that contain G as a spanning subgraph. Finding the threshold strong dimension of a tree T is a challenging problem. We present some results regarding threshold strong dimension of trees. For example, we show that for a starlike tree T with a major vertex of degree less than 9, except for a few cases, the threshold strong dimension $\tau_s(T)$ is equal to 2.

On Three Metric Dimension Variants and the Cyclomatic Number of a Graph

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We consider three metric dimension variants; the classical metric dimension $\dim(G)$, the edge metric dimension $\operatorname{edim}(G)$, and the mixed metric dimension $\operatorname{mdim}(G)$. The cyclomatic number of a graph G (denoted by $c(G)$) is the number of edges one needs to remove from G to obtain a forest, i.e. $c(G) = |E(G)| - |V(G)| + 1$ when G is connected. Sedlar and Škrekovski conjectured in a series of articles that the three metric dimensions can be bounded with the (necessary) number of leaves and the cyclomatic number of the graph; $\dim(G) \le L(G) + 2c(G)$, $\operatorname{edim}(G) \le L(G) + 2c(G)$, $\operatorname{mdim}(G) \le l(G) + 2c(G)$. In 2022, Lu et al. introduced a construction for doubly resolving sets of graphs with minimum degree 2, where the size of the doubly resolving set is at most $2c(G) + 1$. We utilize and extend this construction to prove that the weaker upper bound $L(G) + 2c(G) + 1$ holds for the classical and edge metric dimensions, and that in many cases the stronger bound of the conjecture is attained. Similarly, we prove that $\operatorname{mdim}(G) \le l(G) + 2c(G) + 1$, and that the original conjecture holds in many cases.

On Vertices Belonging to All Metric Bases of G

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Let G be a finite, simple and undirected graph with the set of vertices V and edges E. We say that a nonempty set $S \subseteq V$ is a *resolving set* of G if for all pairs of distinct vertices u and v there exists $s \in S$ such that $d(u, s) \neq d(v, s)$. The smallest cardinality of a resolving set of G is called the *metric dimension* of G and is denoted by $\dim(G)$. Furthermore, a resolving set with $\dim(G)$ vertices is called a *metric basis* of G. Notice that no vertex u belongs to all resolving sets of (a connected and nontrivial graph) G since the set $V \setminus \{u\}$ is clearly resolving. However, this is not the case regarding the metric bases of G. Indeed, there exist graphs G and vertices u such that u belongs to all metric bases of G. This observation motivates the definition of a *basis forced vertex* (of G) which belongs to all metric bases of G.

We show that the problem of deciding whether a vertex is basis forced is co-NP-hard and, hence, an algorithmically demanding problem. Furthermore, we study the existence of basis forced vertices in sparse and dense graphs (concerning the number of edges). In particular, we give bounds for the maximum number of edges in a graph containing basis forced vertices. In addition, particular focus is given to unicyclic graphs for which a partial classification according to the number of basis forced vertices they contain is given.

Distance Formula and Strong Metric Dimension of the Modular Product of Graphs

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The *modular product* $G \diamond H$ of graphs G and H is a graph with a vertex set $V(G) \times V(H)$. Two vertices (g, h) and (g', h') of $G \diamond H$ are adjacent if any of the following conditions hold: (1) $g = g'$ and $hh' \in E(H)$, (2) $gg' \in E(G)$ and $h = h'$, (3) $gg' \in E(G)$ and $hh' \in E(H)$, or (4) for $g \neq g'$ and $h \neq h'$, $gg' \notin E(G)$ and $hh' \notin E(H)$. We derive the distance formula for the modular product and then describe all edges of the strong resolving graph of $G \diamond H$. This information is used to determine the strong metric dimension of the modular product for several infinite families of graphs.

Total mutual-visibility in Hamming graphs

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If G is a graph, then $X \subseteq V(G)$ is a *total mutual-visibility set* of G if every pair of vertices x and y of G is X-visible, that is, there exists a shortest x, y-path P with $V(P) \cap X \subseteq \{x, y\}$. The cardinality of a largest total mutual-visibility set of G is the *total mutual-visibility number* of G. In this talk, we will consider the total mutual-visibility number of Hamming graphs (alias Cartesian products of complete graphs). Exact values will be given for 3-dimensional Hamming graphs. A general upper bound will and a tight asymptotics for balanced Hamming graphs will be presented. The problem will also be reformulated as a Turán-type problem.

On the nonlocal metric dimension of graphs

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Given a connected graph G, a nonlocal resolving set is a set of vertices which resolves each pair of non-adjacent vertices of G. The nonlocal metric dimension $\dim_{n\ell}(G)$ is the cardinality of a smallest nonlocal resolving set of G. This concept was introduced in [1] and further considered in [2]. Several results on this parameter are given in this work. For instance, graphs G with specific values for $\dim_{n\ell}(G)$ are characterized. Also, the nonlocal metric dimension of some product graphs is studied. Additionally, some other combinatorial and computational results are discussed.

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Metric Dimension and Fault-Tolerant Metric Dimension of Generalized Sierpiński Graph

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Covering the nodes in a network, allowing every node to be uniquely identified by looking at the nodes that it is covered by. The minimal number of landmark nodes needed to use graph geodesics to separate any two nodes in the graph is captured by a combinatorial notion known as the metric dimension of a graph. Because they affect how effectively parallel computing systems run on a large scale, interconnection networks are essential. For many interconnection networks, the parameter metric dimension is heavily challenging, as is the fault-tolerant metric dimension too. This paper characterizes the graphs for which the fault-tolerant metric dimension is twice the metric dimension.

Finite metric and k-metric bases on ultrametric spaces

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Given a metric space (X, d), a set $S \subset X$ is called a k-metric generator for X if any pair of different points of X is distinguished by at least k elements of S. A k-metric basis is a k-metric generator of the minimum cardinality in X.

We prove that ultrametric spaces do not have finite k-metric bases for $k > 2$. We also characterize when the metric and 2-metric bases of an ultrametric space are finite and, when they are finite, we characterize them. Finally, we prove that an ultrametric space can be easily recovered knowing only the metric basis and the coordinates of the points in it. This could have applications for the problem of finding a phylogenetic tree or the problem of obtaining a hierarchical clustering for a data set, which is equivalent to giving an appropriate ultrametric to that set.

On the threshold strong dimension of the n-cube

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The n-cube is the graph whose vertex set consists of all binary vectors of length n, with two vertices being adjacent when they differ in precisely one component. While the exact metric dimension of the n-cube is not known, it has been shown to be asymptotically equal to $\frac{2n}{\log_2(n)}$. On the other hand, the strong dimension of the n-cube is precisely 2^{n-1}. The threshold strong dimension of a graph G is the smallest strong dimension among all graphs having G as a spanning subgraph. We show that the threshold strong dimension of the n-cube is surprisingly smaller than its strong dimension. As a tool, we apply a recent theorem that characterizes the threshold strong dimension of a graph in terms of a specific type of embedding into the strong product of paths. Since there are many applications involving metric dimensions of Cartesian products of graphs, one is motivated to generalize our result, leading to some open questions. This is joint work with Nadia Benakli, Novi Bong, Shonda Dueck, Linda Eroh, and Ortrud Oellermann.

On Certain Unary Operations of Graphs and Their Metric Dimension

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A set $W \subset V(G)$ of vertices resolves a graph G if every vertex of G is uniquely determined by its vector distances to the vertices in W. The metric dimension of G is the minimum cardinality of a resolving set. In this paper we discuss metric dimension and fault-tolerant metric dimension of certain unary operations namely double graphs, strong double graphs and splitting graphs.

An Efficient Lower Bound for Edge Metric Dimension and its Fault-Tolerant Version

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A collection of nodes S, in a graph G, is an edge resolving set if every edge of G has a unique vector of distances with respect to S. An edge resolving set with the least possible nodes is an edge metric basis, and its number of vertices is referred as the edge metric dimension, $\dim_E(G)$. A vertex subset F is named fault-tolerant edge metric basis if F has minimum vertices such that $F \setminus \{u\}$ is also an edge resolving set for every $u \in F$. The cardinality of such F is the fault-tolerant edge metric dimension, $\dim'_E(G)$. This paper provides efficient lower bounds for both edge metric dimension and its fault-tolerance of any graph G.

Towards a Characterization of the Metric Dimension of Random Graphs

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Given a graph G, a subset of vertices R is called a resolving set when, for all distinct pairs of vertices u and v, there is at least one $r \in R$ such that $d(r, u) \neq d(r, v)$ where $d(x, y)$ is the shortest path distance between vertices x and y. The metric dimension of G, denoted $\beta(G)$, is the size of smallest resolving sets on the graph. Determining this value exactly for arbitrary graphs is an NP-hard problem. While significant work has been done regarding the metric dimension of specific families of graphs, relatively little is known about the behavior of metric dimension in the context of random graph models. In this talk we will discuss a characterization of the metric dimension of Erdős-Rényi random graphs by Bollobás et al. and present preliminary work related to the metric dimension of graphs from the stochastic block model, Barabási-Albert random graphs, and geometric random graphs. Initial data-driven investiga-

tions have revealed interesting patterns which may serve as the basis of asymptotic bounds and fast heuristic algorithms.

Metric Dimension of a Diagonal Family of Generalized Hamming Graphs

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Given a graph, choose a set of vertices to be a set of landmarks and assign every vertex in the graph a location vector recording the shortest distances to each of the landmarks. If all vertices receive different location vectors, then we say the landmarks resolve the graph. The metric dimension of the graph is the minimum number of landmarks needed to resolve the graph. In this talk, we consider the metric dimension for a diagonal family of 3-dimensional generalized Hamming graphs. The approach is constructive and is made possible by first characterizing resolving sets in terms of forbidden subgraphs of an auxiliary edge-colored hypergraph.

Minisymposium

Structural Graph Theory

Organized by Tony Huynh, *Sapienza University of Rome*
Coorganized by Paul Wollan, *Sapienza University of Rome*

INVITED TALK

Partitioning directed planar digraphs into small diameters.

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We show that for any r and any directed planar graph G, there is a vertex partition $V_1, \dots, V_l$ of $V(G)$ such that for each V_i and for any two vertices x, y of V_i, if there is a path from x to y in the graph induced by V_i. the distance from x to y in G is at most $\log^2 n \times r$.

This implies that the multicommodity flow-cut gap on *directed* planar graphs is $O(\log^3 n)$. This is the first *sub-polynomial* bound for any family of directed graphs of super-constant treewidth (of the abstract graph). We remark that for general directed graphs, it has been shown by Chuzhoy and Khanna in 2009 that the gap is $\Omega(n^{1/7})$, even for directed acyclic graphs.

As a direct consequence of our result, we also obtain the first polynomial-time polylogarithmic-approximation algorithms for the Directed Sparsest-Cut, and the Directed Multicut problems for directed planar graphs, which extends the long-standing result for undirected planar graphs by Rao in 1999 (with a slightly weaker bound). Joint work with Tasos Sidiropoulos.

The structure of directed 1-separations in directed graphs

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At least for small k, there are simple and canonical combinatorial structures displaying all k-separations of a given k-connected undirected graph and the relationships between them. For $k = 1$ this is the block-cut decomposition and for $k = 2$ it is the Tutte decomposition. For directed graphs almost nothing of this kind is known. We will discuss recent progress on uncovering such structure for directed 1-separations.

Pivot-minors and the Erdős-Hajnal conjecture

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Pivot-minors can be thought of as a dense analogue of the graph minor relation. We shall discuss pivot-minors and a recent result that proper pivot-minor-closed classes of graphs have the (strong) Erdős-Hajnal property.

K_6 minors in triangulations with a crossing

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Let T be a triangulation in a closed surface Σ. T is irreducible if every edge e lies in a noncontractible cycle of length 3. The complete sets of irreducible triangulation are only known for surfaces of Euler genus at most 4. Nakamoto et al. have (in a series of papers) exactly characterized all trianguations containing K_6 minors in surfaces for which the complete sets of irreducible triangulations are known. They have conjectured that every 5-connected triangulation of a nonspherical surface Σ contains a K_6 minor.

Let T be a trianguation and t, t' a pair of adjacent triangles. If v, v' are nonadjacent vertices of $V(t \cup t')$ then we say that $T + vv'$ is a graph obtained by *adding a crossing* to T. We show that if T is a 5-connected triangulation in an arbitrary nonspherical surface Σ and T' is obtained from T by adding a crossing, then T' contains a K_6 minor.

Structural theory for odd-minors

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An odd-minor in a graph G is a minor which has a model for which there is a colouring of the vertices in which an edge is monochromatic if and only if it is an edge between two branch sets. Odd-minors have mostly been studied in connection to a variant of Hadwiger's conjecture. We study the structure of graphs excluding certain odd minors and discuss consequences of these structure theorems. This is joint work with Rutger Cambell, Kevin Hendrey, and Sebastian Wiederrecht.

The structure of directed graphs – looking for the missing link

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Excluding a fixed graph as a minor yields a class of graphs that can be decomposed in a tree-like structure such that every bag is embeddable into a surface of bounded genus after removing a bounded set of apex vertex and a bounded number of areas called vortices. This fundamental result by Robertson and Seymour dominates the research in structural graph theory of the last two decades.

Recent research on directed graph structure considers which analogues of minors and grids for directed graphs yield a generalisation of these results. In this talk we take a look at which steps of the structure theory by Robertson and Seymour have been transferred to directed graphs so far. How do these results differ on directed graphs? What about the remaining steps? What are the main difficulties and how can we hope to overcome them?

The presented work contains results by and joint results with Stephan Kreutzer, Ken-ichi Kawarabayashi, Sebastian Wiederrecht.

The Excluded Tree Minor Theorem Revisited

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Robertson and Seymour (GM I) proved that for every tree T there is an integer c such that every T-minor-free graph has pathwidth at most c. Graph product structure theory describes graphs in complicated classes as subgraphs of products of simpler graphs. Inspired by this viewpoint, we prove that for every tree T of radius h, there is an integer c such that every T-minor-free graph is contained in $H \boxtimes K_c$ for some graph H with pathwidth at most $2h-1$. This is a qualitative strengthening of the Excluded Tree Minor Theorem of Robertson and Seymour (GM I). We show that radius is the right parameter to consider in this setting, and $2h-1$ is the best possible bound.

Aharoni's rainbow cycle conjecture holds up to an additive constant

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In 2017, Ron Aharoni proposed the following generalization of the Caccetta-Häggkvist conjecture: if G is a simple n-vertex edge-colored graph with n color classes of size at least r, then G contains a rainbow cycle of length at most $\lceil n/r \rceil$.

I will begin with a summary of recent progress on Aharoni's conjecture based on a new survey article of Katie Clinch, Jackson Goerner, Freddie Illingworth, and myself. I will then sketch a proof that Aharoni's conjecture holds up to an additive constant for each fixed r. The last result is joint work with Patrick Hompe.

Canonical Graph Decompositions via Coverings

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We present a canonical way to decompose finite graphs into highly connected local parts. The decomposition depends only on an integer parameter whose choice sets the intended degree of locality. The global structure of the graph, as determined by the relative position of these parts, is described by a coarser model. This is a simpler graph determined entirely by the decomposition, not imposed.

The model and decomposition are obtained as projections of the tangle-tree structure of a covering of the given graph that reflects its local structure while unfolding its global structure. In this way, the tangle theory from graph minors is brought to bear canonically on arbitrary graphs, which need not be tree-like.

Computing Canonical Graph Decompositions

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The construction of the canonical graph decomposition of a finite graph G into its highly connected local parts is an inherently infinite process. In particular, it builds on the analysis of a covering C of G, which is usually infinite. Nevertheless, we show that this decomposition becomes computable under weak conditions on G and the intended degree of locality.

To implement this, we develop a combinatorial notion of local separations of G which correspond to the separations of C. We then apply recent results from tangle theory to these local separations in order to construct the desired decomposition of G.

Improving the Directed Excluded Grid Theorem

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In 2015, Kawarabayashi and Kreutzer proved the directed grid theorem — the generalization of the well-known excluded grid theorem to directed graphs — confirming a conjecture by Reed, Johnson, Robertson, Seymour, and Thomas from the mid-nineties. The theorem states the existence of a function f such that every digraph of directed tree-width $f(k)$ contains a cylindrical grid of order k as a butterfly minor, but the function grows non-elementarily with the size of the grid minor.

We present an alternative proof of the directed grid theorem which is conceptually much simpler, more modular in its composition and also proves much better bounds. Our proof is inspired by the breakthrough result of Chekuri and Chuzhoy who proved a polynomial bound for the excluded grid theorem for undirected graphs. We translate a key concept of their proof to directed graphs by introducing *cycle-of-well-linked sets (CWS)* and show that any digraph of high directed tree-width contains a large CWS. It is easily seen that a CWS contains a cylindrical grid, this allows us to improve the bounds of Kawarabayashi and Kreutzer's result from a non-elementary to an elementary function.

A direct consequence of our result is an improvement of the bounds in the Erdős–Pósa-property for butterfly minors for those strongly connected directed graphs having that property. A directed graph H has the Erdős–Pósa-property for butterfly minors if there exists a fuction $f : \mathbb{N} \to \mathbb{N}$ such that D contains either k copies of the graph H as a butterfly minor or a set of vertices S of D with at most $f(k)$ elements that meets all copies of H. As the previous result used the directed grid theorem, we can then improve f from a non-elementary to an elementary function.

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Minisymposium

Symmetries Of Graphs And Related Structures

Organized by Isabel Hubard, *National Autonomus University of Mexico*
Coorganized by Primož Šparl, *University of Ljubljana*

INVITED TALK

Characters of twisted fractional linear groups, with applications to regular maps

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For q a prime power, the group $\mathrm{PGL}(2,q)$, well known as the fractional linear group over a finite field of order q, can be obtained as a 2-extension of the group $\mathrm{PSL}(2,q)$ by a diagonal automorphism. If q is an even power of an odd prime, there is another interesting 2-extension of $\mathrm{PSL}(2,q)$ obtained by a composition of a diagonal automorphism with a Frobenius automorphism of order 2. Such extensions, known as *twisted fractional linear groups*, are important in the theory of transitive permutation groups, since by Zassenhaus' classification they form the 'other' infinite family of finite sharply 3-transitive groups (together with the groups $\mathrm{PGL}(2,q)$).

While fractional linear groups and their subgroups have been widely studied, their twisted version have received comparatively less attention. This also applies to characters: despite a number of resources on representations and characters of fractional linear groups, character tables for their 'twisted' version appear to have been neglected.

In our talk we will present character tables for twisted fractional linear groups, including the path towards this result, and indicate its application to enumeration of regular maps of a given type with automorphism group isomorphic to a twisted linear fractional group.

INVITED TALK

Vertex-transitive graphs with large automorphism groups

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Many results in algebraic graph theory can be viewed as upper bounds on the size of the automorphism group of graphs satisfying various hypotheses. These kinds of results have many applications. For example, Tutte's classical theorem on 3-valent arc-transitive graphs led to many other important results about these graphs, including enumeration, both of small order and in the asymptotical sense. This naturally leads to trying to understand barriers to this type of results, namely graphs with large automorphism groups. We will discuss this, especially in the context of vertex-transitive graphs of fixed valency. We will highlight the apparent dichotomy between graphs with automorphism group of polynomial (with respect to the order of the graph) size, versus ones with exponential size.

On stability of graphs –Rose Window graphs

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Let Γ and Σ be two graphs and $\Gamma \times \Sigma$ be the direct product of graphs Γ and Σ. It is easy to see that combing an automorphism of Γ with an automorphism of Σ gives us an automorphism of

$\Gamma \times \Sigma$. Hence, $\mathrm{Aut}(\Gamma) \times \mathrm{Aut}(\Sigma) \leq \mathrm{Aut}(\Gamma \times \Sigma)$. A natural question that arises here is whether $\mathrm{Aut}(\Gamma) \times \mathrm{Aut}(\Sigma) = \mathrm{Aut}(\Gamma \times \Sigma)$. It is hard to answer this question even if the graphs belong to restricted families. We call the graph Γ stable if $\mathrm{Aut}(\Gamma) \times \mathrm{Aut}(\mathrm{K}_2) = \mathrm{Aut}(\Gamma \times \mathrm{K}_2)$, otherwise, it is unstable. We will present some results about unstable families of graphs. In particular, we give a characterization of unstable Rose Window graphs.

New discoveries about regular maps

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In late 2022 Primož Potočnik and I began a programme of extending the list of all orientably-regular maps, from genus 301 up to much larger genus, using some new approaches for finding smooth finite quotients of triangle groups (including the $(2, k, \infty)$ triangle group for certain values of k). Reaching genus 1001 was relatively easy, so then we attempted extending it to genus 1501. This talk will describe the outcome.

Aside from significantly increasing the number of such maps, in both the reflexible and chiral cases, we used the same information to extend the list of all regular non-orientable maps. Along the way we developed new means of determining certain information (such as the edge-multiplicity of each map and its dual), and the resulting lists produced some new information about various issues, such as the relative proportions of reflexible vs chiral maps, solubility of their automorphism groups, and simplicity of the underlying graphs – resolving an open question in the latter case. Also we formulated a nice way of displaying the information, making it more easily searchable.

Chiral polytopes whose smallest regular cover is a polytope

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An abstract polytope $\mathcal{P}$ is *chiral* if the automorphism group has two orbits on the flags such that adjacent flags are always in different orbits. Every chiral polytope has a smallest regular maniplex that covers it. We give a criterion for when this smallest regular cover is itself a polytope, using only information about the facets and vertex-figures of $\mathcal{P}$. We then explore several examples and corollaries.

Towards inductive proofs in algebraic graph theory with applications

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One of the main obstacles to solving problems concerning automorphism groups of vertex-transitive graphs and digraphs is that the natural factors of the automorphism group do not necessarily give groups that are automorphism groups of vertex-transitive graphs or digraphs (or even 2-closed groups). Thus using induction can be difficult. We find a class of transitive permutation groups, which properly contains the automorphism groups of vertex-transitive graphs and digraphs, and call them $5/2$-closed, as, using Wielandt's hierarchy of permutation groups, the class is larger than 2-closed but contain groups which are not 3-closed. We give a sufficient condition for the natural quotients of transitive groups in this class to remain in this class, in effect giving a sufficient condition for natural inductive proofs to work. We will then give examples of results that have been proven for this new class, solving several problems concerning isomorphisms between combinatorial objects, automorphism groups of combinatorial objects, and more.

Some results about the classifications of graphical m-semiregular representation of finite groups

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A graph or digraph is called *regular* if each vertex has the same valency, or the same out-valency and the same in-valency, respectively. Recently, we extend the classical notion of *digraphical* and *graphical regular representation* of a group. A *(di)graphical m-semiregular representation* (respectively, GmSR and DmSR, for short) of a group G is a regular (di)graph whose automorphism group is isomorphic to G and acts semiregularly on the vertex set with m orbits. When $m = 1$, this definition agrees with the classical notion of GRR and DRR. Finite groups admitting a D1SR were classified by Babai in 1980, and the analogue classification of finite groups admitting a G1SR was completed by Godsil in 1981. Pivoting on these two results, we classify finite groups admitting a GmSR or a DmSR (for arbitrary positive integers m) and also do some work about n-partite (di)graphs. This is a joint work with Prof. Yan-Quan Feng and Pablo Spiga.

The Product of a Generalized Quaternion Group And a Cyclic Group

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Let X=QC be a group, where Q is a generalized quaternion group and C is a cyclic group such tha t the intersection of Q and C is trivial. In this paper, X will be characterized and moreover, a complete classification for that will be given, provided C is core-free.

Abstract regular polytopes of hight rank for groups that are neither An nor Sn

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Since 2016 it is known that the rank of a string C-group which is isomorphic to a transitive subgroup of the symmetric group S_n (other than S_n and the alternating group A_n) is at most $n/2 + 1$. Moreover the string C-groups attaining this bound were classified. We aim to extend

this classification down to ranks $n/2$ and $(n-1)/2$.

This is a joint work with Claudio Piedade and Olivia Reade.

On automorphisms of a family of $(q,8)$-graphs

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For given integers $k \geq 2$ and $g \geq 3$ the k-regular graphs of girth g are called (k,g)-*graphs.* The smallest such graphs (with respect to the order) are called (k,g)-*cages.* In the *Cage problem* one has to construct the smallest possible (k,g)-graph, that is, a (k,g)-cage. The $(q+1,8)$-cages are known, when q is an odd prime power. They are arising as incidence graphs of generalized quadrangles, thus they are very symmetric in the sense of automorphisms and transitivity. Natural attempts to obtain small $(q,8)$-graphs from the $(q+1,8)$-cages include also constructions using induced subgraphs. In this talk, maybe surprisingly, we show that a family of such $(q,8)$-graphs of order $2q(q^2-1)$ is not so symmetric in comparison with other such families, showing that their group of automorphisms has precisely four orbits on the set of vertices.

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Neighbour-transitive codes in Kneser graphs

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A code is a subset of the vertex set of a graph. Classically codes have been studied in the Hamming and Johnson graphs; here we study codes in Kneser graphs. The vertex set of a Kneser graph $K(n,k)$ is the set of all k-subsets of an n-set V. Note that if $n = 2k+1$ then $K(n,k)$ is the odd graph O_{k+1}. A code C is s-neighbour-transitive if its automorphism group $\mathrm{Aut}(C)$ acts transitively on each of the sets $C = C_0, C_1, \ldots, C_s$, where $C = C_0, C_1, \ldots, C_\rho$ is the distance partition, and ρ is the covering radius, of C.

First, we give a full classification of 2-neighbour-transitive codes in Kneser graphs with minimum distance at least 5. We then consider neighbour-transitive (i.e., 1-neighbour-transitive) codes in Kneser graphs, giving characterisation results in the cases where $\mathrm{Aut}(C)$ acts intransitively on V and, for the odd graph case, when $\mathrm{Aut}(C)$ acts imprimitively on V. Finally, in the non-odd graph neighbour-transitive case we prove that if C has minimum distance at least 3 and $\mathrm{Aut}(C)$ acts transitively on V then $\mathrm{Aut}(C)$ is in fact 2-homogeneous on V.

Graph Embeddings, Group Factorizations, and Skew Morphisms

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A skew morphism of a finite group A is a permutation φ on A fixing the identity element, and for which there exists an integer-valued function π on A such that $\varphi(xy) = \varphi(x)\varphi^{\pi(x)}(y)$ for all $x, y \in A$. If π is constant on the orbits of φ, the skew morphism φ is called smooth. The theory of skew morphisms is a crucial algebraic tool to investigate symmetric embeddings of graphs into orientable surfaces. In this talk, we will present the most recent progress in the classification problem of skew morphisms of cyclic groups, and show that a cyclic group of order n underlies only smooth skew morphisms if and only if $n = 2^e n_1$, where $0 \leq e \leq 4$ and n_1 is a square-free odd number.

When is Cartesian product a Cayley graph?

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Automorphisms of the Cartesian product of graphs are well understood, and it is known that Cartesian product of Cayley graphs is a Cayley graph. It is natural to ask the reverse question, namely whether all the factors of Cartesian product that is a Cayley graph have to be Cayley graphs. Despite being natural question, it seems to be open, and has not been investigated a lot in the past. The main purpose of this talk is to introduce this problem, demonstrate its complexity, and present certain new results.

Small regular and biregular graphs of prescribed girth constructed as bi-coset graphs

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Given a group G with subgroups H and K, the bi-coset graph constructed from the triple G, H, K is the bipartite graph whose vertices are the (left) cosets of H and K in G with cosets of H connected to cosets of K with which they have a non-empty intersection. While the bi-coset graphs constructed from subgroups of equal orders are necessarily regular, in case of H and K being of different orders, the resulting graphs are bipartite biregular — bipartite having the property that each of the two partition sets consist of vertices of the same degree. The girth of bi-coset graphs is determined by the length of the shortest products constructed from alternating series of elements from H and K that are equal to the identity of G. We show that bi-coset graphs exist of arbitrary large girths and use the bi-coset graph construction to construct small regular or bipartite biregular graphs of large girth. We also investigate connections between graphs obtained in this way and combinatorial designs. The presented results are joint work with Stefan Gyurki, Pavol Janos, Jozef Siran and Yan Wang.

Maps and hypermaps with primitive automorphism groups

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We investigate regular maps and hypermaps with automorphism groups acting primitively on the vertices. In the orientably or fully regular cases, the stabilisers of vertices are respectively cyclic or dihedral. Finite primitive permutation groups with such point stabilisers are known: in the cyclic case they are certain subgroups of the affine group $\mathrm{AGL}_1(q)$, acting on a finite field $\mathbb{F}_q$, and in the dihedral case they are certain subgroups of $\mathrm{A\Gamma L}_1(q)$ and $\mathrm{AGL}_2(\sqrt{q})$, together with almost simple groups $\mathrm{PSL}_2(q)$, $\mathrm{PGL}_2(q)$ and $\mathrm{Sz}(q)$. These results enable us to enumerate and classify the maps and hypermaps associated with each of these group actions. The methods also have applications to other structures, such as dessins d'enfants, Riemann and Klein surfaces, and polytopes.

Girth-biregular graphs, cages and finite geometries

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A new type of graph regularities have been introduced recently. Let Γ denote a simple, connected, finite graph. For an edge e of Γ let $n(e)$ denote the number of girth cycles containing e. For a vertex v of Γ let $\{e_1, e_2, \ldots, e_k\}$ be the set of edges incident to v ordered such that $n(e_1) \leq n(e_2) \leq \cdots \leq n(e_k)$. Then $(n(e_1), n(e_2), \ldots, n(e_k))$ is called the *signature* of v. The graph Γ is said to be *girth-biregular* if it is bipartite, and all of its vertices belonging to the same bipartition have the same signature.

We present several examples and show that girth-biregular graphs are related to biregular cages, finite projective and affine spaces and generalized polygons.

Enumeration of regular maps of given type on twisted linear fractional groups

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We enumerate orientably regular maps of any given type with automorphism group isomorphic to a twisted linear fractional group; these groups form the 'other' family in Zassenhaus' classification of finite sharply 3-transitive groups.

Why hamiltonicity matters?
Infinitely many vertex-transitive graphs with prescribed Hamilton compression

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The following question asked by Lovász in 1970 tying together traversability and symmetry, two seemingly unrelated graph-theoretic concepts, remains unresolved after all these years:

Does every finite connected vertex-transitive graph have a Hamilton path?

In my talk I will discuss certain partial results obtained thus far, specifically a solution to one of the problems traised in [4], together with a connection to another long standing problem regarding vertex-transitive graphs [7].

Is it true that every vertex-transitive graph admits a nontrivial automorphism with all orbits of the same size?

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Automorphisms of direct products of circulant graphs

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For a non-bipartite graph X, the automorphisms of the direct product $X \times K_2$ play an important role in understanding the automorphism group of $X \times Y$, where Y is bipartite. A graph X is unstable if $X \times K_2$ has automorphisms that do not come from automorphisms of its factors. It is non-trivially unstable if it is unstable, connected, non-bipartite and twin-free. We provide new sufficient conditions for the instability of circulant graphs, generalising previously known results. Furthermore, we classify non-trivially unstable members of several families of circulants.

Symmetric Graphs and Polytopes of Even Rank

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In this talk we will explore the relationships between polytopes of even rank and symmetric graphs. In particular, given a polytope $\mathcal{P}$ of rank $2n$, the faces of middle ranks $n-1$ and n

constitute the vertices of a bipartite graph, the medial layer graph $\mathcal{M}(\mathcal{P})$ of $\mathcal{P}$. The group of automorphisms and dualities of $\mathcal{P}$ has a natural action on this graph. We will explore algebraic and combinatorial conditions on $\mathcal{P}$ that ensure this action is transitive on k-arcs in $\mathcal{M}(\mathcal{P})$ for small k, and provide examples of families of polytopes that satisfy these conditions.

Semiregular polytopes with trivial facet stabilizers

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Most of the research on abstract polytopes is done by understanding their symmetries, so it's natural that the most studied polytopes are ones with a los of symmetries. Regular polytopes are by far the most studied ones, and it is known that they can be determined by their groups of symmetries with a given set of generators. Another interesting class are semiregular polytopes: those that are vertex-transitive and have regular facets. Monson and Schulte found a way to construct alternating semiregular polytopes from tail-triangle C-groups, and the examples they construct are hereditary (i.e. the simmetries of each facet induce symmetries of the whole polytope). In this talk we will focus on the oposite of hereditary examples: we are looking for semiregular polytopes with one or two facet orbits such that no symmetry of their facets induce a symmetry of the whole polytope. We will see a way to construct examples with one facet orbit in rank 3 starting from regular polytopes in arbitrary rank. We will also see that the same construction gives 2-facet-orbit examples when applied to a chiral polytope of odd rank. If time allows it, we will talk about how it may be possible to find examples in higher ranks using similar ideas.

Cayley extensions of maniplexes and polytopes

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A map on a surface whose automorphism group has a subgroup acting regularly on its vertices is called a Cayley map. Cayley extensions are a generalisation of that notion to maniplexes and polytopes. We define M to be a *Cayley extension* of K if the facets of M are isomorphic to K and if some subgroup of the automorphism group of M acts regularly on the facets of M. We will show that many natural extensions in the literature on maniplexes and polytopes are in fact Cayley extensions. We sill also describe several universal Cayley extensions and examine the possibilities for their symmetry type graph.

Towards a classification of CI$^{(3)}$-groups

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A finite group R is called a CI$^{(k)}$-group if any two k relational Cayley structures over R are isomorphic iff they are isomorphic by an automorphism of R. An intensive study of CI$^{(2)}$-groups had started with Ádám's conjecture posed in 1967. In 1977 Babai proposed a group theoretical approach to a classification of CI$^{(k)}$-groups for arbitrary $k \geq 2$. Using this approach P.Pálfy classified all CI$^{(k)}$-groups with $k \geq 4$ in 1987. While a classification of CI$^{(2)}$-groups was a very popular topic, CI$^{(3)}$-groups were rather untouched till 2003 when E. Dobson started to investigate the problem seriously. In my talk I'll report the recent results regarding classification of CI$^{(3)}$-groups.

Joint work with Edward Dobson and Pablo Spiga.

$\acute{A}d\acute{a}m$'s Conjecture and the Isomorphisms of Circulant Graphs

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A circulant graph, $C_n(a_1, \ldots, a_k)$, on n vertices is one that has each vertex v adjacent to vertex $(v \pm a_i) \mod n$. A natural question is to determine which two circulant graphs are isomorphic. Let $S = \{a_i\}_{i=1}^{k}$. $\acute{A}d\acute{a}m$ conjectured that $G(n, S) \cong H(n, S')$ if and only if there exists an $x \in \mathbb{Z}_n^*$ such that $S' = xS$; this is called the Cayley isomorphism property. Elspas and Turner disproved this for $n = p^2$. Later, Muzychuk answered the general isomorphism classification question for circulant graphs using existence arguments. In this talk, we present a constructive approach that explicitly defines *all* the isomporphisms which determine the isomorphism classes of circulant graphs not only those conjectured by $\acute{A}d\acute{a}m$.

Title: Designs, graphs, and their q-analogues.

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Abstract: Let t, λ, k, and v be positive integers such that $\lambda < k-1 < v-1$, A $t-(v, k, \lambda)$-block design is an incidence structure $(P, \mathcal{B})$, where P is a set of v points, and $\mathcal{B}$ is a set of k-subsets of points (called blocks), such that every t-subset of P is contained in exactly λ blocks. Now let q be a prime power. A q-analogue of a $t - (v, k, \lambda)$ design (also called a subspace design or a $t - (v, k, \lambda)_q$ design) is an incidence structure $(V, \mathcal{B})$ where V is a vector space of dimension v over the field $GF(q)$, $\mathcal{B}$ is a collection of subspaces of V of dimension k each, and such that every subspace of dimension t of V is contained in exactly λ elements of $\mathcal{B}$ (k-subspaces). Similarly, q-analogues of graphs and other incidence structures can be defined, changing the notions of (sub)sets to (sub)spaces, and cardinality to dimension. In this talk we will give a brief overview of this topic.

Core-free degrees of Toroidal Chiral (Hyper)Maps

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Every group G can be represented as a faithful transitive permutation representation of degree n. Moreover, the stabilizer of a point in this permutation representation is always a core-free subgroup of G. Conversely, the action of a group G on a core-free subgroup $H \leq G$ is always transitive and faithful, giving a faithful transitive permutation representation on the set of cosets G/H, with degree $|G : H|$. These permutation representations are powerful tools in the classification of abstract regular/chiral polytopes and hypertopes.

Following previous results on regular structures, in this talk we will focus on the chiral case, listing all possible degrees of faithful transitive permutation representations of the toroidal chiral maps $\{4, 4\}$, $\{3, 6\}$ and hypermaps $(3, 3, 3)$. This is a joint work with M. Elisa Fernandes.

Symmetry breaking by arc colouring of digraphs

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An edge coloring of a graph is distinguishing if the only automorphism of the graph that preserves the coloring is the identity. The smallest number of colors in a proper distinguishing coloring of a graph G is called its chromatic distinguishing index. Kalinowski and Pilśniak proved in 2015 that always $\Delta(G) + 1$ colors is enough to break all non-trivial automorphisms of a connected graph of order at least 7.

We will consider various definitions of proper colorings of the arcs of symmetric digraphs. We will show upper bounds of the chromatic index in each case, and the distinguishing chromatic index of a connected symmetric digraph with respect to the maximal degree of its underlying graph. We will justify the optimality of the obtained constraints.

In the talk, we survey results on asymmetric edge colourings of graphs. We give known general upper bounds in terms of maximum degree. We also formulate some conjectures in this topic.

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Saxl graphs

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A *base* for a permutation group $G \leq \mathrm{Sym}(\Omega)$ is a subset B of Ω whose pointwise stabiliser in G is trivial. The *base size* of G is the minimal cardinality of a base for G. Small bases are of interest for computational reasons, and there is a particular focus in the literature on permutation groups with base size 2. Given such a group, one can define its *Saxl graph*, with vertex set Ω and two vertices being adjacent if and only if they comprise a base. Saxl graphs were introduced by Burness and Giudici, who conjectured that if G is finite and primitive then its Saxl graph has the property that every two vertices have a common neighbour. We present some evidence towards this conjecture, and describe various open cases which pose interesting problems in computational group theory, representation theory of finite simple groups, and additive combinatorics.

Exploring Rose Window graphs and their connection with tetraciculant cubic graphs

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In this talk we explore in detail some of the interesting properties and substructures of one of the most important known family of arc-transitive tetravalent graphs. Known as the Rose Window graphs, these graphs were first introduced by S. Wilson in 2008 and have since undergone extensive study, leading to a comprehensive understanding. During the presentation, we establish a connection between these tetravalent graphs and the classification of tetraciculant cubic graphs through the "split graph" construction. By doing so, we show which of these cubic graphs are tetracilculant and provide some of their properties, as well their corresponding diagrams. The results presented are joint work with Steve Wilson, Micael Toledo and Primoz Potocnik.

On bi-coset digraphs and their automorphism groups

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Bi-coset graphs, which were originally defined by Du and Xu in 2000, completely characterize all semi-transitive graphs and can be viewed as generalizations of Haar (di-)graphs (sometimes called bi-Cayley (di-)graphs). In this talk, we will find strong constraints on the automorphism groups of bi-coset graphs and digraphs.

Skeletal Snub Polyhedra in Ordinary Space

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Skeletal polyhedra are discrete structures consisting of finite (planar or skew) or infinite (linear, zigzag, or helical) polygons as faces, with two faces on each edge and a circular vertex-figure at each vertex. We describe the blueprint for the snub construction and show that it can be applied to both regular and chiral skeletal polyhedra. The resulting snub polyhedra are vertex-transitive and highly locally symmetric. Their properties - from a combinatorial, topological, and geometric perspective - are described. We discuss the completeness of our list of generated structures and examine when the construction yields uniform skeletal polyhedra.

Maniplexes that are pretty, but not that pretty.

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Coauthor: Dimitri Leemans

Maniplexes are coloured graphs that generalise maps on surfaces and abstract polytopes. An interesting class of maniplexes that possess two properties, called faithfulness and thinness, are particularly close to abstract polytopes. All faithful thin maniplexes of rank three are known to be isomorphic to the flag-graph of an abstract polytope. Until recently, only one example of rank four of a thin maniplex that is not isomorphic to the flag-graph of an abstract polytope was known. In this talk we discuss the concepts of faithfulness, thinness and polytopality and we show that there are infinitely many faithful thin maniplexes that are not polytopal for every rank $n > 3$. In short, we talk about maniplexes that are pretty, but not that pretty.

On trialities and their absolute geometries

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Coauthors: Dimitri Leemans, Klara Stokes

We introduce the notion of moving absolute geometry of a geometry with triality and show that, in the classical case where the triality is of type (I_σ) and the absolute geometry is a generalized hexagon, the moving absolute geometry also gives interesting flag-transitive geometries with Buekenhout diagram $(5, 3, 6)$ for the groups $G_2(k)$ and ${}^3D_4(k)$, for any integer $k \geq 2$. If times permits, we will also look at classical and moving absolute geometries of some Class III maps that were contructed by D.Leemans and K.Stokes.

A family of chiral polyhedra with helical faces in $\mathbb{R}^4$.

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Chiral geometric polyhedra have been little studied. In $\mathbb{R}^3$ there aren't finite chiral polyhedra, but there are three infinite families (which was proved by Egon Schulte). There are few examples of chiral geometric polyhedra in other spaces.

In this talk, we will classify the chiral polyhedra with helical faces whose skeleton is contained in the skeleton of the regular convex 4-polytopes in $\mathbb{R}^4$.

A Wythoff's type construction for chiral polyhedra

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In this talk we shall review how some convex polyhedra can be generated using the classical Wythoff's construction, and how this construction has been modify to give skeletal polyhedra. We then address the notion of a chiral polytope and present a Wythoff-type construction to obtain them. It will be shown how the Wythoff construction is a useful tool to understand the symmetry and structure of these geometric objects.

Motions of articulated structures using graphs of groups

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In structural rigidity, one studies frameworks of bars and joints in Euclidean space. Such a framework is an articulated structure consisting of rigid bars, joined together at joints around which the bars may rotate. I will present a model of articulated motions of realisations of incidence geometries that uses the terminology of graph of groups, and describe the motions of such a framework using group theory. Our approach allows to model a variety of situations, such as parallel redrawings, scenes, polytopes, realisations of graphs on surfaces, and even unique colourability of graphs. This approach allows a concise description of various dualities in rigidity theory. We also provide a lower bound on the dimension of the infinitesimal motions of such a framework in the special case when the underlying group is a Lie group. This is joint work with Klara Stokes.

Some remarks on realizations of polyhedra and polytopes

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We give an overview of the classification of geometric polyhedra with high degree of symmetry, and discuss the realization theory of abstract regular polytopes.

On infinite 2-orbit polyhedra

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In 1977 Grünbaum introduced skeletal polyhedra, in which a polyhedron is a skeleton (embedding of a graph) equipped with distinguished subgraphs that play the role of facets. Grünbaum identified 47 regular skeletal polyhedra, and Dress found a 48th and proved that the resulting list was complete in 1981 and 1985.

A flag of a polyhedron is an incident vertex, edge, and facet. Regular polyhedra are those whose automorphism group acts transitively on the flags. Fully-transitive polyhedra have automorphism groups that act transitively on the vertices, edges, and facets, and so include the regular polyhedra, but allow for other symmetry types. This talk will be concerned with our efforts to classify the infinite polyhedra whose flags fall into two orbits under the actions of the symmetry group of the polyhedron.

All McKay-Miller-Širáň graphs are lifts of dipoles

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Inspired by the degree-diameter problem, McKay, Miller and Širáň introduced a family of large and highly symmetric graphs H_q, where q is a prime power greater than two. These graphs are of diameter 2, order $2q^2$ and degree $(3q - r)/2$, where q is congruent to r mod 4. Moreover, McKay-Miller-Širáň (MMS) graphs H_q with $q = 3$, $q = 4$ and $q = 4k + 1$ are vertex transitive but not Cayley and for other values of q the automorphism group splits the vertices into two orbits.

In the original paper, the MMS graphs are constructed as lifts of complete bipartite graphs $K_{q,q}$. Šiagiová later managed to simplify this construction for graphs H_q with $q = 4k + 1$. Her voltage construction uses a base graph with only two vertices, i.e. a dipole. In this talk we show that all MMS graphs can be constructed as a lift of a dipole. During the proof we use results originating from Hafner, who described all MMS graphs geometrically by using incidence graphs of finite affine planes and who also completely determined automorphism groups of all MMS graphs.

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